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Module 9: Single-level and Multilevel Models for Ordinal Responses

Stata Practical¹

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Pre-requisites

- Modules 5, 6 and 7

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¹ This Stata practical is adapted from the corresponding MLwiN practical: Steele, F. (2011). Single-level and Multilevel Models for Ordinal Responses - Stata Practical. LEMMA VLE Module 9, 1-48. Accessed at <http://www.bristol.ac.uk/cmm/learning/course.html>

Introduction to the Eurobarometer 2009 Dataset on Interest in EU Elections

You will be analysing data from the Eurobarometer Opinion and Social Questionnaire from spring 2009.² The analysis sample contains residents of the 29 European Union Member States³ who were aged 15 years and over, selected using a multi-stage probability design.

Our response variable is an ordinal indicator of the level of interest in European elections. Respondents were asked:

The next European elections will be held in June 2009. How interested or disinterested would you say you are in these elections?

and presented with the following response alternatives: Very interested, Somewhat interested, Somewhat disinterested, Very disinterested, and Don't know.

After excluding the small number of "don't knows" and respondents from candidate EU states who were not asked this question, the sample size is 26,126. For purposes of illustration, and to speed up model estimation, we take a 50% sample and exclude a small percentage of individuals with missing values on any of the explanatory variable considered. The analysis sample contains 10,340 individuals with the sample size for each state ranging from 98 to 509. The data therefore have a two-level hierarchical structure with individuals at level 1, nested within states at level 2.

We consider several predictor variables. The dataset contains only individual-level variables, but we will derive state-level aggregates for consideration as level 2 predictors. The individual-level variables are gender, age, occupation type, and an index of left-right political attitudes.⁴

The file contains the following variables:

Variable name	Description and codes
state	EU state identifier

² Eurobarometer 71.1: European Parliament and Elections, Economic Crisis, Climate Change, and Chemical Products, January-February 2009 (Study No. ZA4971). Go to <http://www.gesis.org/en/eurobarometer-data-service/> for further information on the Eurobarometer series and to download datasets.


³ The survey was also conducted in the three candidate countries (Croatia, Turkey and Macedonia) and in the Turkish Cypriot Community, but they are not included in our analysis file because the response variable (interest in EU elections) was not available for respondents in these countries.

⁴ Respondents were asked to rate their political views on a 10-point scale in response to the question: "In political matters people talk of 'the left' and 'the right'. How would you place your views on this scale?"

person	Individual identifier
electint	Interest in EU elections (1=very low, 2=low, 3=some, 4=very high) ⁵
female	Individual gender (1=female, 0=male)
agecen50	Individual age in years (centred at 50)
agecen50sq	Individual age in years (centred at 50) squared
occtype	Occupation type (1=manager, 2=other employed, 3=looking after home/family, 4=unemployed, 5=retired, 6=student)
lrplace	Placement on scale of left-right political attitudes (a 10-point scale with high values indicating more right wing views)
commtype	Type of community of residence (1=rural, 2=mid-sized town, 3=large town or city)

Load “9.1.dta” into memory and open the do-file “9.1.do” for this lesson.

From within the LEMMA Learning Environment

- Go to **Module 9: Single-Level and Multilevel Models for Ordinal Responses**, and scroll down to  **Stata datasets and dofiles**
- Click “9.1.dta” to open the dataset

Use the `summarize` command to view the variables in the dataset:

```
. summarize
```


Variable	Obs	Mean	Std. Dev.	Min	Max
state	10,340	14.81576	8.942563	1	30
person	10,340	5170.5	2985.045	1	10340
electint	10,340	2.455222	.9038433	1	4
female	10,340	.5267892	.499306	0	1
agecen50	10,340	-.6407157	17.60089	-35	48
agecen50sq	10,340	310.1721	323.4805	0	2304
occtype	10,340	3.234816	1.620245	1	6
lrplace	10,340	5.292843	2.307556	1	10
commtype	10,340	1.904449	.794336	1	3

⁵ The coding of the original variable was reversed so that high values indicate greater interest. ‘Very high’ corresponds to ‘very interested’, ‘some’ to ‘somewhat interested’, ‘low’ to ‘somewhat disinterested’, and ‘very low’ to ‘very disinterested’.

P9.1 Cumulative Logit Model for Single-Level Data

Load “9.1.dta” into memory, and if it is not already in use open the do-file “9.1.do” for this lesson.

From within the LEMMA Learning Environment

- Go to **Module 9: Single-Level and Multilevel Models for Ordinal Responses**, and scroll down to  **Stata datasets and dofiles**. Click “9.1.dta” to open the dataset

P9.1.1 Specifying and estimating and cumulative logit model

We will begin by examining the distribution of our response variable, level of interest in EU elections. Use the `tabulate` command to view the number (`Freq.`) and percentage (`Percent`) of respondents in each response category

```
. tabulate electint
```

Interest in European elections	Freq.	Percent	Cum.
vlow	1,773	17.15	17.15
low	3,255	31.48	48.63
some	4,144	40.08	88.70
vhigh	1,168	11.30	100.00
Total	10,340	100.00	

The percentage in each of the four response category is shown. The cumulative response percentages, working upwards from the ‘very low’ category are 17.2%, 48.6%, 88.7%, 100%⁶.

Our first model will simply reproduce the cumulative probabilities, from which we can derive the response probabilities. The model is a single-level ordered logistic regression with no covariates. Let $y_i = s$ denote the ordinal response for respondent i ($i = 1, \dots, n$) where $s = 1, 2, 3, 4$ denotes the four response categories “vlow”, “low”, “some” and “vhigh”. The model can then be written as

$$\text{logit}\{\Pr(y_i > s | x_{1i})\} \equiv \log \left\{ \frac{\Pr(y_i > s)}{1 - \Pr(y_i > s)} \right\} = -\kappa_s, \quad s = 1, 2, 3$$

⁶ Note that the `ologit` and `meologit` estimation commands for fitting single-level and multilevel ordinal response models cumulate the response category probabilities the other way around.

where the only parameters to be estimated are the three cut points κ_1 , κ_2 and κ_3 .

We fit the above model using the `ologit` command. The model converges after one iteration:

```
. ologit electint
```

```
Iteration 0:   log likelihood = -13224.823
Iteration 1:   log likelihood = -13224.823
```

```
Ordered logistic regression               Number of obs   =       10,340
                                          LR chi2(0)       =           0.00
                                          Prob > chi2      =           .
Log likelihood = -13224.823               Pseudo R2       =       0.0000
```

electint	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
/cut1	-1.575245	.026091			-1.626382 -1.524107
/cut2	-.0549461	.0196759			-.0935101 -.0163822
/cut3	2.060862	.0310675			1.999971 2.121754

The first cut point `/cut1` is estimated to be -1.575 and tells us that the log-odds of having low, some or very high interest in EU elections ($s > 1$) relative to very low interest ($s = 1$) is 1.575. This corresponds to a probability of having low, some or very high interest in EU elections of $\exp(1.575)/[1+\exp(1.575)] = 0.828$. It follows that the probability of having instead very low interest in EU elections is simply $1 - 0.828$ or 0.172.

The second cut point `/cut2` is estimated to be -0.055 and so the the log-odds of having some or very high interest in EU elections is 0.055, which corresponds to a probability of 0.514. The probability of having instead very low or low interest in EU elections is $1 - 0.514$ or 0.486.

Finally, the third cut point `/cut3` is estimated to be 2.061 and so the log-odds of having very high interest in EU elections is -2.061 which corresponds to a probability of 0.113. The probability of having instead very low, low or some interest in EU elections is $1 - 0.113$ or 0.887.

Reassuringly, these probabilities all agree with the cumulative percentages from our earlier tabulation of `electint`.

We could have carried out these calculations using Stata's post estimation `predict` command to calculate the predicted probability for each category of `electint`.

```
. predict p*
(option pr assumed; predicted probabilities)
```

Stata generates four new variables `p1`, `p2`, `p3` and `p4` which store, for each respondent, the predicted probability of each response category. We can use the `summarize` command to display summary statistics of the predictions:

```
. summarize p1-p4
```

Variable	Obs	Mean	Std. Dev.	Min	Max
p1	10,340	.17147	0	.17147	.17147
p2	10,340	.3147969	0	.3147969	.3147969
p3	10,340	.4007737	0	.4007737	.4007737
p4	10,340	.1129594	0	.1129594	.1129594

The model includes no covariates and so the predicted probabilities are the same for all 10,340 respondents. The predicted probabilities from the model match the response category percentages reported in the earlier one-way tabulation of **electint**. We can also obtain the cumulative probabilities presented in that tabulation by summing the category-specific probabilities appropriately. We do this by generating a new variable for each cumulative probability using the `generate` command:

```
. generate p12 = p1 + p2
. generate p123 = p1 + p2 + p3
. generate p1234 = p1 + p2 + p3 + p4
```

Summarizing these new variables gives the cumulative response probabilities:

```
. summarize p1 p12 p123 p1234
```

Variable	Obs	Mean	Std. Dev.	Min	Max
p1	10,340	.17147	0	.17147	.17147
p12	10,340	.4862669	0	.4862669	.4862669
p123	10,340	.8870406	0	.8870406	.8870406
p1234	10,340	1	0	1	1

These values 0.171, 0.486 and 0.887 agree with our earlier one-way tabulation of **electint**. Finally, we remove all these newly generated variables from the dataset using the `drop` command:

```
. drop p1-p1234
```

P9.1.2 Adding gender

We will next allow for gender differences in election interest, but before including gender in our model we look at a tabulation of **electint** by **female**. Use the `tabulate` command with the option `row` to display row percentages alongside cell and row and column total frequencies:

```
. tabulate female electint, row
```

Key					
frequency					
row percentage					
Interest in European elections					
female	vlow	low	some	vhigh	Total

0		765	1,463	2,036	629		4,893
		15.63	29.90	41.61	12.86		100.00
1		1,008	1,792	2,108	539		5,447
		18.51	32.90	38.70	9.90		100.00
Total		1,773	3,255	4,144	1,168		10,340
		17.15	31.48	40.08	11.30		100.00

There is some suggestion that women tend to have less interest in EU elections than men. For example, 12.86% of men have very high interest compared to 9.90% of women.

We will now include **female** as an explanatory variable in our ordered logistic regression model to see whether this effect is statistically significant. The model is written as.

$$\text{logit}\{\Pr(y_i > s | x_{1i})\} \equiv \log \left\{ \frac{\Pr(y_i > s)}{1 - \Pr(y_i > s)} \right\} = \beta_1 \text{female}_i - \kappa_s, \quad s = 1, 2, 3$$

Fitting the model produces the following results.

```
. ologit electint female
```

```
Iteration 0:   log likelihood = -13224.823
Iteration 1:   log likelihood = -13202.978
Iteration 2:   log likelihood = -13202.971
Iteration 3:   log likelihood = -13202.971
```

```
Ordered logistic regression           Number of obs   =      10,340
                                      LR chi2(1)        =       43.70
                                      Prob > chi2       =       0.0000
Log likelihood = -13202.971          Pseudo R2       =       0.0017
```

electint		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
female		-.2383992	.0360969	-6.60	0.000	-.3091478 -.1676506
/cut1		-1.705829	.0329324			-1.770376 -1.641283
/cut2		-.1813667	.0275056			-.2352768 -.1274567
/cut3		1.940385	.0358646			1.870092 2.010679

Notice that the term **female** appears once in the results window as it applies equally to all three log-odds contrasts. This is because by default Stata fits a proportional odds model that assumes the effect of **female** on the log-odds of being in a higher category of **electint** is the same whether we fix the higher category at 'low', 'some' or 'very high'. (We will relax this assumption later).

The negative estimate for the coefficient of **female** indicates that women are less likely than men to be in a higher category of **electint**⁷. The estimate is much larger

⁷ The magnitude of the female coefficient is the same as that obtained in the MLwiN practical but the coefficient is of the opposite sign. This difference results from the baseline category in the MLwiN practical being set to "very high" to avoid computational problems which arise if the default of "very low" is used instead. As a result, the interpretation of all the models in the MLwiN practical are the

than its standard error (0.036) resulting in a z-ratio of $z = -6.60$ and so the gender difference is statistically significant. To compute predicted probabilities of **electint** by gender, we use Stata's `predict` command again:

```
. predict p*
(option pr assumed; predicted probabilities)
```

To view the predictions for each gender separately, we now combine the `summarize` command with the `bysort` command. The `bysort` command tells Stata to sort on one variable (in our case, **female**) and report the summary statistics for each category of this variable separately:

```
. bysort female: summarize p1 p2 p3 p4
```

```
-----
-> female = 0
```

Variable	Obs	Mean	Std. Dev.	Min	Max
p1	4,893	.1537055	0	.1537055	.1537055
p2	4,893	.3010767	0	.3010767	.3010767
p3	4,893	.4196123	0	.4196123	.4196123
p4	4,893	.1256055	0	.1256055	.1256055

```
-----
-> female = 1
```

Variable	Obs	Mean	Std. Dev.	Min	Max
p1	5,447	.1873335	0	.1873335	.1873335
p2	5,447	.3269207	0	.3269207	.3269207
p3	5,447	.3840739	0	.3840739	.3840739
p4	5,447	.1016719	0	.1016719	.1016719

Note that the probabilities broadly match those reported in a previous two-way tabulation of **female** and **electint**, suggesting that the proportional odds assumption is reasonable. We can obtain the corresponding cumulative probabilities in the same way as before:

```
. generate p12 = p1 + p2
. generate p123 = p1 + p2 + p3
. generate p1234 = p1 + p2 + p3 + p4
. bysort female: summarize p1 p12 p123 p1234
```

```
-----
-> female = 0
```

Variable	Obs	Mean	Std. Dev.	Min	Max
p1	4,893	.1537055	0	.1537055	.1537055

otherway around. They are in terms of the log-odds of being in a lower category of **electint**. The `ologit` and `melogit` commands do not run into the same computational problems and so here we retain Stata's default of having "very low" as the baseline category. In sum, the interpretation of all models in this Stata practical is in terms of the log-odds of being in a higher category and all regression coefficients will be of the same magnitude but the opposite sign to those reported in the MLwiN practical.

```

p12 |      4,893      .4547822      0      .4547822      .4547822
p123 |      4,893      .8743945      0      .8743945      .8743945
p1234 |      4,893      1      0      1      1

```

```
-----
-> female = 1
```

```

Variable |      Obs      Mean      Std. Dev.      Min      Max
-----+-----
p1 |      5,447      .1873335      0      .1873335      .1873335
p12 |      5,447      .5142542      0      .5142542      .5142542
p123 |      5,447      .8983281      0      .8983281      .8983281
p1234 |      5,447      1      0      1      1

```

In the table below, the predicted cumulative response probabilities are shown alongside the observed probabilities (based on the response probabilities from our earlier cross-tabulation). The two sets of probabilities would be exactly the same if we had not assumed that the gender effect was proportional. Their similarity therefore again suggests that the proportional odds assumption is reasonable, but we will now test this more formally.

Electint	Gender	Cumulative response probability	
		Predicted	Observed
Very low	Male	0.154	0.156
<= low	Male	0.455	0.455
< = some	Male	0.874	0.871
Very low	Female	0.187	0.185
<= low	Female	0.514	0.514
< = some	Female	0.898	0.901

Finally, drop the predicted probability variables from the dataset:

```
. drop p1-p1234
```

P9.1.3 Testing the proportional odds assumption

So far we have assumed that the effect of gender on the log-odds of being in election interest category greater than k is the same wherever we fix k ⁸. For example, we assume that the gender difference in the log-odds of having *very high* interest is the same as the gender difference in the log-odds of having *very high* or *high* interest. We can test this proportional odds assumption by allowing the coefficient of **female** to vary according to which response category we are considering. This model, commonly referred to as a *generalised ordered logit model*⁹, cannot be fitted using `ologit` but can be fitted through a user written command named `gologit2`. First,

⁸ Note that the interpretation of the model in Stata is different to that in the MLwiN practical. This is due to differences model parameterisations between the two programmes whereby in Stata the model is parameterised in terms of the log-odds of being in category $k+1$ or higher vs. k or lower, while in MLwiN the model is parameterised in terms of the log-odds of being in category $k-1$ or lower vs. k or higher.

⁹ See Cole, S.R., Allison, P.D. and Ananth, C.V. (2004) Estimation of cumulative odds ratios. *Annals of Epidemiology*, 14: 172-178.

we need to install this command from the Boston College Statistical Software Components (SSC) archive¹⁰:

```
. ssc install gologit2
checking gologit2 consistency and verifying not already installed...
all files already exist and are up to date.
```

To more clearly demonstrate the change that relaxing the proportional odds assumption makes, we will first re-run the previous proportional odds model using `gologit2` with the `pl` option (shorthand for “parallel lines”):

```
. gologit2 electint female, pl
```

Generalized Ordered Logit Estimates

Number of obs	=	10,340
LR chi2(1)	=	43.70
Prob > chi2	=	0.0000
Pseudo R2	=	0.0017

Log likelihood = -13202.971

```
( 1) [vlow]female - [low]female = 0
( 2) [low]female - [some]female = 0
```

		electint	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
vlow							
	female		-.2383992	.0360969	-6.60	0.000	-.3091478 -.1676506
	_cons		1.705829	.0329324	51.80	0.000	1.641283 1.770376
low							
	female		-.2383992	.0360969	-6.60	0.000	-.3091478 -.1676506
	_cons		.1813667	.0275056	6.59	0.000	.1274567 .2352768
some							
	female		-.2383992	.0360969	-6.60	0.000	-.3091478 -.1676506
	_cons		-1.940386	.0358646	-54.10	0.000	-2.010679 -1.870092

We now store the estimates of this model using Stata’s `estimates` command:

```
. estimates store pom
```

The log-likelihood of -13202.971 is identical to that reported by the `ologit` command confirming that both commands are fitting the same model. Note, however, that while the estimated cut-points given by each command are identical in absolute magnitude, they differ in their signs. This is due to the different way the two models are parameterised. In the `ologit` command the cut points are subtracted from the linear predictor, while in the `gologit` command they are added. Thus, the `gologit` parameterisation of the model is

$$\text{logit}\{\Pr(y_i > s | x_{1i})\} \equiv \log \left\{ \frac{\Pr(y_i > s)}{1 - \Pr(y_i > s)} \right\} = \beta_1 \mathbf{female}_i + \kappa_s, \quad s = 1, 2, 3$$

¹⁰ The Boston College SSC archive is the largest collection of user-written Stata programs for data manipulation, statistics, and graphics

Looking again at the output of the `gologit` command, we can see that the proportional odds assumption is being imposed in this model as the coefficient of **female** is constrained to be equal across the three log-odds contrasts. We will now relax the proportional odds assumption. The resulting model is written as

$$\text{logit}\{\Pr(y_i > s | x_{1i})\} \equiv \log \left\{ \frac{\Pr(y_i > s)}{1 - \Pr(y_i > s)} \right\} = \beta_{s,1} \mathbf{female}_i + \kappa_s, \quad s = 1, 2, 3$$

where we have added an *s* subscript to the regression coefficient to indicate that it can now vary across the three log-odds contrasts. We fit this model by removing the `pl` option from the `gologit` command:

```
. gologit2 electint female
```

Generalized Ordered Logit Estimates	Number of obs	=	10,340
	LR chi2(3)	=	45.22
	Prob > chi2	=	0.0000
Log likelihood = -13202.215	Pseudo R2	=	0.0017

electint		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
vlow							
	female	-.2032115	.0526002	-3.86	0.000	-.3063059	-.1001171
	_cons	1.685672	.0393629	42.82	0.000	1.608523	1.762822
low							
	female	-.2352922	.0394841	-5.96	0.000	-.3126797	-.1579048
	_cons	.1790997	.0287066	6.24	0.000	.1228359	.2353636
some							
	female	-.2950745	.0623168	-4.74	0.000	-.4172132	-.1729358
	_cons	-1.913832	.0427124	-44.81	0.000	-1.997546	-1.830117

```
. estimates store npom
```

We can now see that the coefficient of **female** now varies across the three log-odds contrasts. The estimated coefficients of **female** are not greatly different, but nevertheless suggest that the gender difference is smallest for the odds of being above the very low category and largest for the odds of being above the some category.

If the proportional odds assumption holds, the three **female** coefficients will be equal to each other. We can test for non-proportionality using a Likelihood Ratio test¹¹:

```
. lrtest pom npom, force
```

Likelihood-ratio test	LR chi2(2)	=	1.51
(Assumption: pom nested in npom)	Prob > chi2	=	0.4692

¹¹ In the MLwiN practical we use a Wald test instead of a Likelihood Ratio test, because likelihood values that are required for Likelihood Ratio tests are not available from the MLwiN models. To run a Wald test in Stata you can use the `test` command:

```
. test ([v]low|female - [low]female = 0) ([low]female - [some]female = 0)
```

The Likelihood ratio test statistic is 1.51 which we compare with a chi-squared distribution on 2 degrees of freedom. The associated p -value is 0.469 so we cannot reject the null hypothesis that the three female coefficients are equal, and we conclude that the proportional odds assumption holds. We will therefore revert to the model with a common coefficient for **female**.

P9.1.4 Adding further explanatory variables

We will next allow for age and occupation effects on election interest. The age variable (**agecen50**) has been centred around 50 years, which affects only the threshold estimates. To fit a quadratic age effect we also include the age squared variable (**agecen50sq**). Occupation (**occtype**) is a nominal variable with six categories. We enter these as five dummy variables, omitting the first category

$$\begin{aligned} \text{logit}\{\Pr(y_i > s | \mathbf{x}_{ij})\} \\ = \beta_1 \mathbf{female}_i + \beta_2 \mathbf{agecen50}_i + \beta_3 \mathbf{agecen50sq}_i + \beta_4 \mathbf{otherwork}_i \\ + \beta_5 \mathbf{home}_i + \beta_6 \mathbf{unemployed}_i + \beta_7 \mathbf{retired}_i + \beta_8 \mathbf{student}_i - \kappa_s, \\ s = 1, 2, 3 \end{aligned}$$

When running analyses in Stata using categorical variables, these must be prefixed by **i.** so that Stata knows to treat the variable as categorical and specify series of dummy variables in the model as appropriate:

```
. ologit electint female agecen50 agecen50sq i.occtype
```

```
Iteration 0:   log likelihood = -13224.823
Iteration 1:   log likelihood = -13085.516
Iteration 2:   log likelihood = -13085.233
Iteration 3:   log likelihood = -13085.233
```

```
Ordered logistic regression               Number of obs   =    10,340
                                          LR chi2(8)      =    279.18
                                          Prob > chi2     =    0.0000
Log likelihood = -13085.233              Pseudo R2      =    0.0106
```

electint	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
female	-.2567263	.037169	-6.91	0.000	-.3295762	-.1838765
agecen50	.0136195	.0016324	8.34	0.000	.01042	.016819
agecen50sq	-.0003498	.0000685	-5.11	0.000	-.000484	-.0002155
occtype						
Other_work	-.4335095	.060301	-7.19	0.000	-.5516973	-.3153218
Home	-.3124214	.0909148	-3.44	0.001	-.4906112	-.1342316
Unemployed	-.8216828	.0895783	-9.17	0.000	-.9972529	-.6461126
Retired	-.5470199	.0771501	-7.09	0.000	-.6982314	-.3958084
Student	-.0042884	.1059398	-0.04	0.968	-.2119265	.2033497
/cut1	-2.266661	.062842			-2.389829	-2.143493
/cut2	-.7167323	.0588767			-.8321285	-.6013362
/cut3	1.435172	.0615437			1.314549	1.555795

Starting with the effects of **occtype**, the negative coefficients for all five **occtype** dummies indicate that respondents in these categories have lower interest in EU elections than the reference category of Managers, although the effect of Student is negligible and not statistically significant. Unemployed people are least interested in EU elections.

Turning to the effect of age, we find that both the linear and quadratic terms are statistically significant. The positive linear age effect combined with the negative squared age effect suggest that the probability of being in a higher interest category increases with age, but flattens off (or perhaps even starts to decrease) at older ages. Alternatively, we can say that younger people tend to be less interested in EU elections than older people. To examine the nature of the quadratic age effect, we will calculate and plot predicted cumulative probabilities for election interest by age. The easiest way to do this is to first refit the model using manually created dummy variables for the variable **occtype** rather than relying on the **i.** prefix. We can generate these dummy variables by specifying the **generate()** option of the **tabulate** command.

```
. tabulate occtype, generate(o)
```

Occupation type	Freq.	Percent	Cum.
Manager	1,175	11.36	11.36
Other_work	4,053	39.20	50.56
Home	691	6.68	57.24
Unemployed	665	6.43	63.68
Retired	3,102	30.00	93.68
Student	654	6.32	100.00
Total	10,340	100.00	

The command will name the six dummy variables **o1**, **o2**, **o3**, **o4**, **o5** and **o6**.

We now rerun the model with these manually created occupation type dummies. Note that we exclude the category **o1** in order to force this as the baseline category for the **occtype** variable:

```
. ologit electint female agecen50 agecen50sq o2-o6
```

```
Iteration 0:  log likelihood = -13224.823
Iteration 1:  log likelihood = -13085.516
Iteration 2:  log likelihood = -13085.233
Iteration 3:  log likelihood = -13085.233
```

Ordered logistic regression	Number of obs	=	10,340
	LR chi2(8)	=	279.18
	Prob > chi2	=	0.0000
Log likelihood = -13085.233	Pseudo R2	=	0.0106

electint	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
female	-.2567263	.037169	-6.91	0.000	-.3295762 -.1838765
agecen50	.0136195	.0016324	8.34	0.000	.01042 .016819
agecen50sq	-.0003498	.0000685	-5.11	0.000	-.000484 -.0002155
o2	-.4335095	.060301	-7.19	0.000	-.5516973 -.3153218
o3	-.3124214	.0909148	-3.44	0.001	-.4906112 -.1342316

o4	-.8216828	.0895783	-9.17	0.000	-.9972529	-.6461126
o5	-.5470199	.0771501	-7.09	0.000	-.6982314	-.3958084
o6	-.0042884	.1059398	-0.04	0.968	-.2119265	.2033497
-----+-----						
/cut1	-2.266661	.062842			-2.389829	-2.143493
/cut2	-.7167323	.0588767			-.8321285	-.6013362
/cut3	1.435172	.0615437			1.314549	1.555795
-----+-----						

As expected, the results are identical to the model where we relied on the `i.` prefix to automatically generate these dummy variables.

In order to obtain predicted probabilities for age effects only (holding all other variables constant), we need to fix the **female** and **occtype** variables in our model at their mean values. To do this quickly we use a `foreach` loop that iterates over these covariates and performs the same series of commands on each variable. Here the commands are to first `summarize` the variable to display its mean value and then `replace` to fix each covariate at its mean value. In terms of the latter, `r(mean)` is Stata's shorthand way of referring to the mean reported in the previous `summarize` command. You will need to run the series of four commands in one go:

```
. foreach var of varlist female o2 o3 o4 o5 o6 {
.   summarize `var'
.   replace `var' = r(mean)
. }
```

Variable	Obs	Mean	Std. Dev.	Min	Max
female	10,340	.5267892	.499306	0	1

variable female was byte now float
(10,340 real changes made)

Variable	Obs	Mean	Std. Dev.	Min	Max
o2	10,340	.3919729	.4882143	0	1

variable o2 was byte now float
(10,340 real changes made)

Variable	Obs	Mean	Std. Dev.	Min	Max
o3	10,340	.0668279	.2497357	0	1

variable o3 was byte now float
(10,340 real changes made)

Variable	Obs	Mean	Std. Dev.	Min	Max
o4	10,340	.0643133	.2453222	0	1

variable o4 was byte now float
(10,340 real changes made)

Variable	Obs	Mean	Std. Dev.	Min	Max
o5	10,340	.3	.4582797	0	1

variable o5 was byte now float
(10,340 real changes made)

Variable	Obs	Mean	Std. Dev.	Min	Max
o6	10,340	.0632495	.243423	0	1

variable o6 was byte now float
(10,340 real changes made)

Now that each covariate has been replaced with its mean value we can compute the category specific probabilities of election interest:

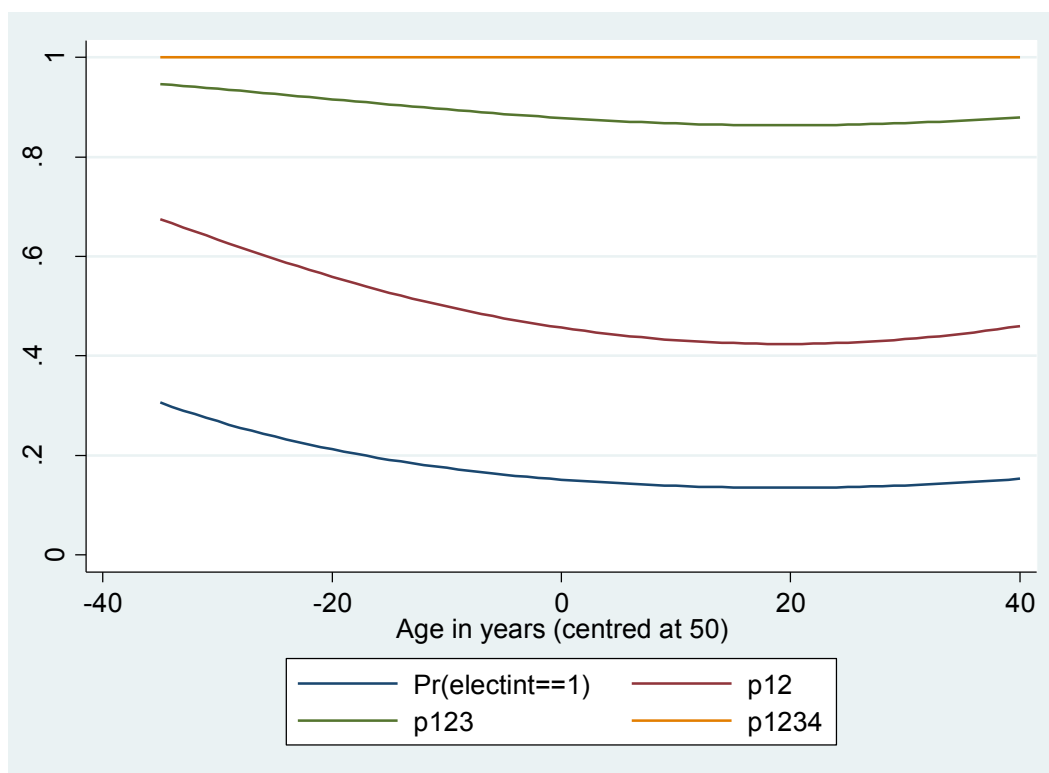
```
. predict p*
(option pr assumed; predicted probabilities)
```

As before, we obtain the cumulative probabilities by cumulating these category specific probabilities:

```
. generate p12 = p1 + p2
. generate p123 = p1 + p2 + p3
. generate p1234 = p1 + p2 + p3 + p4
```

We now create a line graph of these cumulative probabilities by age using the `twoway` command. We use the option `sort` to sort the data so that the resulting line plots connect the data points in the correct order, the `if` qualifier `inrange()` to select only individuals who are within the age range -35 to 40 (i.e., 15 to 90 because age is centred at 50), and the `ylabel` option to format the values on the y-axis to range from 0 to 1 in increments of 0.2. The command is very long and so we have spread it over multiple lines using the line continuation syntax `///`. You will need to run all six lines in one go to issue the command:

```
. twoway ///
  (line p1 agecen50, sort) ///
  (line p12 agecen50, sort) ///
  (line p123 agecen50, sort) ///
  (line p1234 agecen50, sort) ///
  if inrange(agecen50,-35,40), ylabel(0(.2)1)
```


The graph shows four curves, one for how each cumulative response probability depends on age. On the log-odds scale, the curves would be parallel because we are assuming that age effects are proportional. Each curve shows that the probability of being in a lower interest category declines with age, although the decline is small at older ages (as indicated by the ‘flattening off’ of the curve). Younger people tend to be less interested in EU elections than older people.

P9.2 Continuation Ratio Model

In this exercise, we show how to apply the continuation ratio model to educational transitions in Stata. We will reproduce the results given in Table 9.14 of C9.2.

Load “9.2.dta” into memory and if it is not already in use open the do-file “9.2.do” for this lesson.

From within the LEMMA Learning Environment

- Go to **Module 9: Single-Level and Multilevel Models for Ordinal Responses**, and scroll down to  **Stata datasets and dofiles** Click “9.2.dta” to open the dataset

Use the `summarize` command to view the variables in the dataset:

```
. summarize
```

Variable	Obs	Mean	Std. Dev.	Min	Max
degree	979	1.506639	1.217085	0	4
fhs	979	.1532176	.3603812	0	1
female	979	.5617978	.4964199	0	1

The dataset contains three variables; **degree**, **fhs** and **female**. The outcome variable is the respondent's highest level of education, **degree**, which has five ordered categories (less than high school, high school, junior college, Bachelors, and graduate). To see the distribution of **degree** use the `tabulate` command:

```
. tabulate degree
```

RS Highest	Freq.	Percent	Cum.
Degree			
< HS	165	16.85	16.85
HS	499	50.97	67.82
Jnr college	60	6.13	73.95
Bachelor	164	16.75	90.70
Graduate	91	9.30	100.00
Total	979	100.00	

The other variables, **fhs** and **female**, are indicators for whether the father's level of education was greater than high school and gender. These will be included as predictors of the respondent's education level. Some restructuring of the data is required before estimating the continuation ratio model. We need to get the data into the form shown in Table 9.13 in C9.2, but with four transitions rather than three. To do this we need to reshape the dataset from wide to long format using a series of commands. First, generate a unique identifier variable named **person** that runs from 1 to n:

```
. generate person = _n
```

We now need to duplicate the data to give four rows of data per person in order to examine the transitions. We first use the `order` command to make **person** the first variable in the dataset. We then use the `expand` command to duplicate each existing row of data four times:

```
. order person
. expand 4
(2,937 observations created)
```

Now that the data has been expanded, we create a new variable for each possible educational transition by combining Stata's `bysort` and `generate` commands. We tell Stata to sort on **person** and then for each person generate a variable called **trans** with a number from 1 to 4 for each row of data the person has in the dataset:

```
. bysort person: generate trans = _n
```

We now use the `order` command to make **person** and **trans** the first variables in the dataset. Next we generate a new binary variable named **resp** to indicate if an individual made the educational transition for any given transition (row of data) to

a higher level of education. Last we `drop` all unnecessary observations (transitions) from the dataset:

```
. order person trans

. generate resp = (trans<=degree)

. drop if trans > degree + 1
(1,553 observations deleted)
```

We are now ready to generate dummy variables for each transition in the dataset:

```
. tabulate trans, gen(t)
```

trans	Freq.	Percent	Cum.
1	979	41.43	41.43
2	814	34.45	75.88
3	315	13.33	89.21
4	255	10.79	100.00
Total	2,363	100.00	

Finally we need to use a `forvalues` loop to generate new versions of the `fhs` and `female` variables that are specific to each transition `t`:

```
. forvalues t = 1/4 {
.   generate t`t'Xfhs = t`t'*fhs
.   generate t`t'Xfem = t`t'*fem
. }
```

At this point it is helpful to inspect the restructured data. We use the `list` command to do this, restricting the output to the first five individuals:

```
. list person degree trans resp t1-t4 fem fhs if person<=5, sepby(person) nolabel
```

	person	degree	trans	resp	t1	t2	t3	t4	female	fhs
1.	1	3	1	1	1	0	0	0	0	0
2.	1	3	2	1	0	1	0	0	0	0
3.	1	3	3	1	0	0	1	0	0	0
4.	1	3	4	0	0	0	0	1	0	0
5.	2	3	1	1	1	0	0	0	1	0
6.	2	3	2	1	0	1	0	0	1	0
7.	2	3	3	1	0	0	1	0	1	0
8.	2	3	4	0	0	0	0	1	1	0
9.	3	1	1	1	1	0	0	0	1	0
10.	3	1	2	0	0	1	0	0	1	0
11.	4	4	1	1	1	0	0	0	1	0
12.	4	4	2	1	0	1	0	0	1	0
13.	4	4	3	1	0	0	1	0	1	0
14.	4	4	4	1	0	0	0	1	1	0
15.	5	1	1	1	1	0	0	0	0	0
16.	5	1	2	0	0	1	0	0	0	0

Person 1 has four records, and so was at risk of making all four transitions. However, he did not make the fourth transition from Bachelors to Graduate because **resp**=0 for **trans**=4. (We know person 1 is male because **female** is equal to zero). You can see that the data structure resembles that in Table 9.13 in C9.2.

We are now ready to specify the model. The model is a single-level logistic regression of the variable **resp** on the 12 newly created covariates for the intercept, **fhs**, and **female** for each transition.

$$\begin{aligned} \text{logit}\{\text{Pr}(y_i = 1|\mathbf{x}_i)\} &\equiv \log\left\{\frac{\text{Pr}(y_i = 1|\mathbf{x}_i)}{1 - \text{Pr}(y_i = 1|\mathbf{x}_i)}\right\} \\ &= \beta_1 \mathbf{t1}_i + \beta_2 \mathbf{t1}_i \times \mathbf{fhs}_i + \beta_3 \mathbf{t1}_i \times \mathbf{fem}_i + \beta_4 \mathbf{t2}_i + \beta_5 \mathbf{t2}_i \times \mathbf{fhs}_i \\ &\quad + \beta_6 \mathbf{t2}_i \times \mathbf{fem}_i + \beta_7 \mathbf{t3}_i + \beta_8 \mathbf{t3}_i \times \mathbf{fhs}_i + \beta_9 \mathbf{t3}_i \times \mathbf{fem}_i + \beta_{10} \mathbf{t4}_i \\ &\quad + \beta_{11} \mathbf{t4}_i \times \mathbf{fhs}_i + \beta_{12} \mathbf{t4}_i \times \mathbf{fem}_i \end{aligned}$$

We fit this model using the **logit** command where we specify the option **nocons** to suppress the constant term from the model:

```
. logit resp t1 t1xfhs t1xfem t2 t2xfhs t2xfem t3 t3xfhs t3xfem ///
  t4 t4xfhs t4xfem, nocons
```

```
Iteration 0:   log likelihood = -1637.9068
Iteration 1:   log likelihood = -1247.6643
Iteration 2:   log likelihood = -1236.3105
Iteration 3:   log likelihood = -1235.5193
Iteration 4:   log likelihood = -1235.5108
Iteration 5:   log likelihood = -1235.5108
```

```
Logistic regression               Number of obs   =       2,363
                                Wald chi2(12)      =       517.12
Log likelihood = -1235.5108      Prob > chi2    =       0.0000
```

resp	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
t1	1.262392	.1278629	9.87	0.000	1.011785	1.512999
t1xfhs	2.920894	.7175387	4.07	0.000	1.514544	4.327244
t1xfem	.2616396	.1741564	1.50	0.133	-.0797006	.6029798
t2	-.5133183	.1182419	-4.34	0.000	-.7450682	-.2815683
t2xfhs	1.527348	.1970377	7.75	0.000	1.141162	1.913535
t2xfem	-.4241056	.1519498	-2.79	0.005	-.7219218	-.1262895
t3	1.283936	.233553	5.50	0.000	.8261805	1.741691
t3xfhs	1.470975	.4237006	3.47	0.001	.6405371	2.301413
t3xfem	-.3223875	.2974069	-1.08	0.278	-.9052943	.2605194
t4	-.431916	.2113229	-2.04	0.041	-.8461013	-.0177307
t4xfhs	-.0268581	.2725474	-0.10	0.921	-.5610411	.507325
t4xfem	-.3148056	.2646546	-1.19	0.234	-.833519	.2039079

The estimates and standard errors agree with the results in Table 9.14 in C9.2.

In C9.2 a test for equality of the father's education effect across the four transitions was carried out using a Wald test. The null hypothesis for this test that the four interaction coefficients between time and fathers education are all equal can be conducted in Stata as follows:

```
. test (t1xfhs=t2xfhs) (t2xfhs=t3xfhs) (t3xfhs=t4xfhs)
```

```
( 1)  [resp]t1xfhs - [resp]t2xfhs = 0
( 2)  [resp]t2xfhs - [resp]t3xfhs = 0
( 3)  [resp]t3xfhs - [resp]t4xfhs = 0

      chi2( 3) =    28.96
Prob > chi2 =    0.0000
```


The Wald test statistic is 28.96 which is compared to a chi-squared distribution on 3 d.f. This is highly significant ($p < 0.001$), so we reject the null and conclude that the effect of father's education differs across transitions.

P9.3 Random Intercept Cumulative Logit Model

We will now continue our analysis of interest in EU elections using cumulative logit models, but extending our earlier single-level analysis to account for state effects on election interest.

Load “9.3.dta” into memory and if it is not already in use open the do-file “9.3.do” for this lesson.

From within the LEMMA Learning Environment

- Go to **Module 9: Single-Level and Multilevel Models for Ordinal Responses**, and scroll down to  **Stata datasets and dofiles**. Click “9.3.dta” to open the dataset

P9.3.1 Specifying and estimating a simple two-level model

We begin by fitting a two-level variance-components ordinal logistic regression model. The model can be written as

$$\text{logit}\{\Pr(y_i > s)\} = u_j - \kappa_s, \quad s = 1, 2, 3$$

$$u_j \sim N(0, \sigma_u^2)$$

where u_j is a normally distributed state random effect with mean zero and variance σ_u^2 , a model parameter to be estimated.

We use the `meologit` command specifying `||` to declare which variables are to be treated as fixed effects (left hand side of the double separator) and which are to be treated as random effects (right hand side of the double separator). The first variable to the left hand side must be the response variable itself and the first variable to the right hand side of the double separator must be the cluster variable, followed by a colon. In this very first model, the fixed-part of the model includes only an intercept, while the random-part includes only a random-intercept effect.

```
. meologit electint || state:
```

```
Fitting fixed-effects model:
```

```
Iteration 0:    log likelihood = -13224.823
```

```
Iteration 1:    log likelihood = -13224.823
```

```
Refining starting values:
```

```
Grid node 0:    log likelihood = -13019.876
```

```
Fitting full model:
```

```
Iteration 0:    log likelihood = -13019.876    (not concave)
```

```
Iteration 1:    log likelihood = -13015.25    (not concave)
```

```
Iteration 2:    log likelihood = -13010.922
```

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```

Iteration 3:   log likelihood = -13007.135
Iteration 4:   log likelihood = -13003.839
Iteration 5:   log likelihood = -13003.837
Iteration 6:   log likelihood = -13003.837

Mixed-effects ologit regression
Group variable:      state

Number of obs      =      10,340
Number of groups   =         29

Obs per group:
    min =          98
    avg =        356.6
    max =         509

Integration method: mvaghermite
Integration pts.   =          7

Log likelihood = -13003.837
chi2()             =          .
Prob > chi2        =          .
-----+-----
electint |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-----+-----
    /cut1 |   -1.662047   .0929283   -17.89  0.000   -1.844183   -1.479911
    /cut2 |   -.0872315   .0912433    -0.96  0.339   -.2660651    .0916021
    /cut3 |    2.102987   .0943406    22.29  0.000    1.918082    2.287891
-----+-----
state     |
var(_cons)|    .2278971   .0645282             .1308351    .3969661
-----+-----
LR test vs. ologit model: chibar2(01) = 441.97      Prob >= chibar2 = 0.0000

```

The estimated between-state variance is 0.228. By default `meologit` uses mean-variance adaptive Gauss-Hermite quadrature with 7 integration points for each set of random effects. Increasing the number of integration points will lead to a more accurate approximation of the log likelihood and model parameters but requires more computing time. One should keep increasing the number of integration points until the estimated parameters stop changing. Thus, in order to explore whether the default of 7 integration points is sufficient for the current model, we will re-run the model but specify 15 integration points using the `intpoints()` option:

```

. meologit electint || state:, intpoints(15)

Fitting fixed-effects model:

Iteration 0:   log likelihood = -13224.823
Iteration 1:   log likelihood = -13224.823

Refining starting values:

Grid node 0:   log likelihood = -13019.876

Fitting full model:

Iteration 0:   log likelihood = -13019.876 (not concave)
Iteration 1:   log likelihood = -13009.827
Iteration 2:   log likelihood = -13004.235
Iteration 3:   log likelihood = -13003.84
Iteration 4:   log likelihood = -13003.837
Iteration 5:   log likelihood = -13003.837

Mixed-effects ologit regression
Group variable:      state

Number of obs      =      10,340
Number of groups   =         29

Obs per group:
    min =          98
    avg =        356.6

```

```

max = 509
Integration method: mvaghermite      Integration pts. = 15
Log likelihood = -13003.837          chi2() = .
                                      Prob > chi2 = .
-----+-----
electint |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-----+-----
    /cut1 |   -1.662047   .0929283   -17.89   0.000   -1.844183   -1.479911
    /cut2 |   -.0872315   .0912433    -0.96   0.339    -.266065    .091602
    /cut3 |    2.102987   .0943406    22.29   0.000    1.918083    2.287891
-----+-----
state    |
var(_cons) |   .2278969   .0645282                .130835    .3969657
-----+-----
LR test vs. ologit model: chibar2(01) = 441.97      Prob >= chibar2 = 0.0000

```

The log likelihood and parameter estimates from this model using 15 integration points are effectively the same as that from the earlier model, and therefore we can conclude that in this case the default setting of 7 integration points is sufficient and provides an accurate approximation. We therefore return to the original `meologit` command:

```
. meologit electint || state:
```

P9.3.2 Interpretation of the null two-level model

In the above model there is a state-level random effect u_j included. From the estimates above, the prediction equations for the log-odds of being in a particular response category or lower are:

$$\text{Log odds of low, some or very high interest} = u_j + 1.662$$

$$\text{Log odds of some or very high interest} = u_j + 0.087$$

$$\text{Log odds of very high interest} = u_j - 2.103$$

The addition of u_j to each equation allows the cumulative response probabilities to vary across states. As in any multilevel model, we can obtain estimates of the u_j from which we can compute state-specific probabilities.

One way to assess the size of state effects is to compute the variance partition coefficient (VPC). The VPC is defined in C9.3.1 and is interpreted as the proportion of the total (residual) variance in the underlying propensity to have high interest in EU elections that is attributable to differences between EU states. This underlying propensity is the latent continuous response variable that underlies the observed ordered variable (see C9.1.5). The between-state variance is estimated as 0.228 which implies a VPC of $0.228 / (0.228 + 3.29) = 0.065$, so approximately 6.5% of the variation in EU election interest is due to between-state variation.

We will now examine estimates of the state effects or residuals \hat{u}_j obtained from the null model. To calculate the residuals and produce a ‘caterpillar plot’ with the state effects shown in rank order together with 95% confidence intervals we first use Stata’s `predict` command with the `reffects` option to calculate empirical Bayes predictions (a.k.a., posterior means or shrinkage estimates) of the random effects in our model. We specify the `reses` option to obtain the associated standard errors. We store the predicted values in a new variable `u` and their standard errors in a new variable `use`:

```
. predict u, reffects reses(use)
(calculating posterior means of random effects)
(using 7 quadrature points)
```

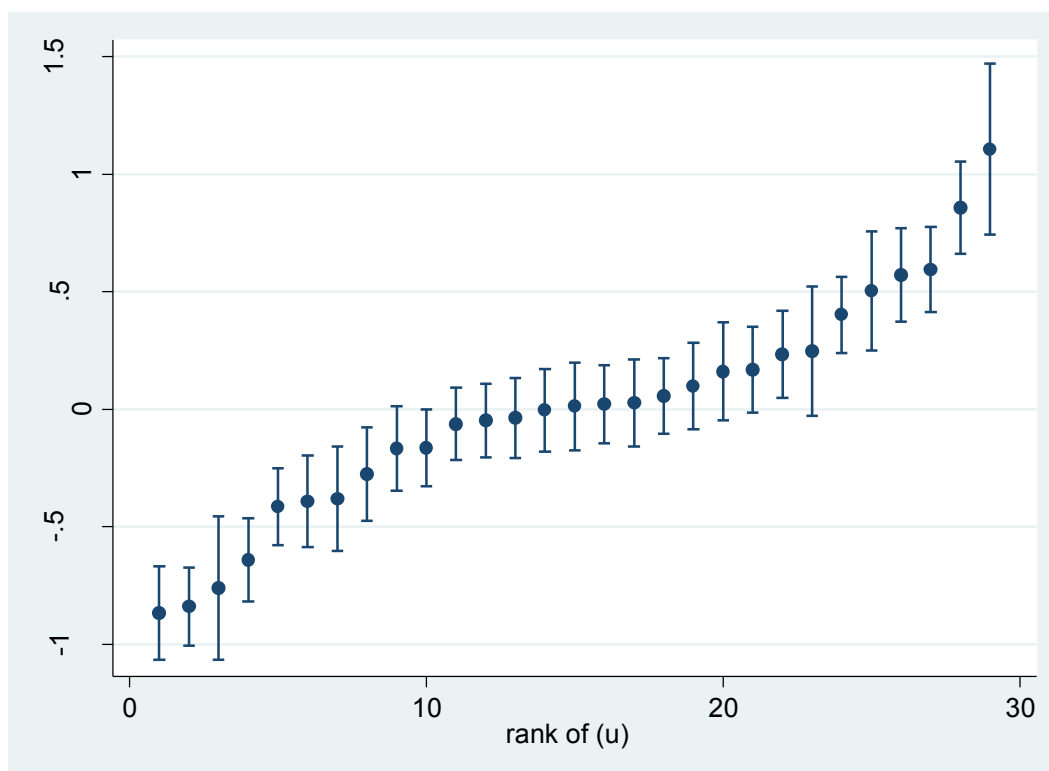
Now that we have predictions of the random effects, we can create a binary indicator `pickone` that identifies each state in the dataset once (states appear multiple times but we only want one record of each state in order to produce the caterpillar plot), and then a variable named `urank` that ranks each state based upon its predicted value:

```
. egen pickone = tag(state)

. egen urank = rank(u) if pickone==1
```

Finally we use the `serrbar` command to create the caterpillar plot using the predicted value and standard error for each state in rank order. We specify `1.96` in the `scale()` option to request 95% confidence intervals:

```
. serrbar u use urank, scale(1.96)
```



The plot shows the estimated residuals for the 29 EU states¹². For a substantial number of states, the 95% confidence interval does not overlap zero, indicating that interest in EU elections is significantly above average (above the zero line) or below average (below the zero line) at the 5% level for these states.

To determine which residual refers to which state we use the `sort` command to sort the dataset from the smallest predicted state effect to the largest:

```
. sort urank
```

We can now display the rank, name and predicted effect for each state:

```
. list urank state u if pickone==1
```

	urank	state	u
1.	1	Latvia	-.8670277
2.	2	Czech Republic	-.8409024
3.	3	Northern Ireland	-.7608606
4.	4	Great Britain	-.6421201
5.	5	Slovakia	-.4154112
6.	6	Poland	-.3920325
7.	7	Germany East	-.381467
8.	8	Portugal	-.2755389
9.	9	Estonia	-.1674033
10.	10	Belgium	-.1645108
11.	11	Sweden	-.0628576
12.	12	Denmark	-.0472776
13.	13	France	-.0368419
14.	14	Austria	-.0045329
15.	15	Italy	.0117315
16.	16	Germany West	.0198793
17.	17	Spain	.0268569
18.	18	Finland	.0569717
19.	19	Slovenia	.0987055
20.	20	Lithuania	.1610961
21.	21	Hungary	.1681147
22.	22	Bulgaria	.2338549
23.	23	Cyprus (Republic)	.2465879
24.	24	The Netherlands	.4013712
25.	25	Luxembourg	.5033886
26.	26	Romania	.5715325
27.	27	Greece	.5947767
28.	28	Ireland	.8567677
29.	29	Malta	1.107149

We can see that Malta has the highest rank and therefore the greatest interest in EU elections. The confidence interval for Malta is wider than for other states, which is probably because it has fewer respondents in the sample. To see the sample size in each state:

¹² These residuals are the same but of the opposite sign to those in the MLwiN module due to the differing parameterisations used by Stata and MLwiN

```
. tabulate state
```

EU state ID	Freq.	Percent	Cum.
France	450	4.35	4.35
Belgium	461	4.46	8.81
The Netherlands	480	4.64	13.45
Germany West	459	4.44	17.89
Italy	340	3.29	21.18
Luxembourg	198	1.91	23.09
Denmark	490	4.74	27.83
Ireland	363	3.51	31.34
Great Britain	429	4.15	35.49
Northern Ireland	132	1.28	36.77
Greece	388	3.75	40.52
Spain	373	3.61	44.13
Portugal	317	3.07	47.20
Germany East	241	2.33	49.53
Finland	465	4.50	54.02
Sweden	509	4.92	58.95
Austria	408	3.95	62.89
Cyprus (Republic)	174	1.68	64.57
Czech Republic	429	4.15	68.72
Estonia	368	3.56	72.28
Hungary	396	3.83	76.11
Latvia	295	2.85	78.97
Lithuania	286	2.77	81.73
Malta	98	0.95	82.68
Poland	310	3.00	85.68
Slovakia	446	4.31	89.99
Slovenia	378	3.66	93.65
Bulgaria	353	3.41	97.06
Romania	304	2.94	100.00
Total	10,340	100.00	

We can see that the sample size for Malta is 98, much lower than for other states.

P9.3.3 Adding explanatory variables

The next step of our analysis is to include some explanatory variables. We will add **female**, **agecen50** (and its square **agecen50sq**) and **occtype** to give the multilevel extension of the single-level model fitted in P9.1.4. The model can be written as

$$\begin{aligned} \text{logit}\{\Pr(y_i > s | \mathbf{x}_{ij})\} \\ = \beta_1 \text{female}_{ij} + \beta_2 \text{agecen50}_{ij} + \beta_3 \text{agecen50sq}_{ij} + \beta_4 \text{otherwork}_{ij} \\ + \beta_5 \text{home}_{ij} + \beta_6 \text{unemployed}_{ij} + \beta_7 \text{retired}_{ij} + \beta_8 \text{student}_{ij} + u_j - \kappa_s, \\ s = 1, 2, 3 \\ u_j \sim N(0, \sigma_u^2) \end{aligned}$$

Note that because the explanatory variables are to be entered as fixed effects they are specified in the `meologit` command to the left of the double separator `||`:

```
. meologit electint female agecen50 agecen50sq i.occtype || state:
```

Fitting fixed-effects model:

```
Iteration 0:   log likelihood = -13224.823
Iteration 1:   log likelihood = -13085.516
```

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```
Iteration 2:   log likelihood = -13085.233
Iteration 3:   log likelihood = -13085.233
```

Refining starting values:

```
Grid node 0:   log likelihood = -12893.392
```

Fitting full model:

```
Iteration 0:   log likelihood = -12893.392 (not concave)
Iteration 1:   log likelihood = -12883.273
Iteration 2:   log likelihood = -12874.274
Iteration 3:   log likelihood = -12870.868
Iteration 4:   log likelihood = -12870.814
Iteration 5:   log likelihood = -12870.814
```

```
Mixed-effects ologit regression
Group variable:      state
```

```
Number of obs      =    10,340
Number of groups   =         29
```

```
Obs per group:
      min =         98
      avg =       356.6
      max =        509
```

```
Integration method: mvaghermite
```

```
Integration pts.   =         7
```

```
Log likelihood = -12870.814
```

```
Wald chi2(8)       =       263.27
Prob > chi2        =       0.0000
```

electint	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
female	-.2111279	.0376188	-5.61	0.000	-.2848594	-.1373965
agecen50	.0133066	.0016531	8.05	0.000	.0100666	.0165465
agecen50sq	-.0003684	.0000692	-5.32	0.000	-.000504	-.0002328
occtype						
Other_work	-.4696381	.0613232	-7.66	0.000	-.5898293	-.3494469
Home	-.598897	.0942797	-6.35	0.000	-.7836817	-.4141122
Unemployed	-.8112878	.0910196	-8.91	0.000	-.989683	-.6328926
Retired	-.5561512	.0784995	-7.08	0.000	-.7100074	-.4022949
Student	-.0259284	.1070556	-0.24	0.809	-.2357535	.1838966
/cut1	-2.377667	.1103936	-21.54	0.000	-2.594034	-2.161299
/cut2	-.7742671	.1080291	-7.17	0.000	-.9860001	-.562534
/cut3	1.452576	.1093899	13.28	0.000	1.238176	1.666977
state						
var(_cons)	.2311513	.0655831			.1325532	.4030903

```
LR test vs. ologit model: chibar2(01) = 428.84      Prob >= chibar2 = 0.0000
```


Comparing the effects of the explanatory variables for the single-level model of P9.1.4 and the above multilevel model, we find that the substantive conclusions are unaffected by allowing for state effects. However, the multilevel model allows us to explore the state-level variation, for example by allowing the effects of one or more predictor to vary across states (in a random slopes model). We can also examine contextual effects on election interest, i.e. the effects of state-level predictors. We will pursue these lines of enquiry in the following lessons.

P9.4 Random Slope Cumulative Logit Model

In the previous lesson we allowed interest in elections to vary across EU states using a random intercept model. We also allowed for election interest to depend on several individual characteristics. However, the effects of these characteristics were assumed to be the same in each state. We now consider random slope models in which the coefficients of one or more explanatory variables can vary from state to state.

Load “9.4.dta” into memory and if it is not already in use open the do-file “9.4.do” for this lesson.

From within the LEMMA Learning Environment

- Go to **Module 9: Single-Level and Multilevel Models for Ordinal Responses**, and scroll down to  **Stata datasets and dofiles** Click “9.4.dta” to open the dataset

First we will refit the previous random-intercept model. Recall that the model includes three covariates: **gender**, **age** (with linear and quadratic terms) and **occupation type** (represented by five dummy variables):

```
. meologit electint female agecen50 agecen50sq i.occtype || state:
```

Fitting fixed-effects model:

```
Iteration 0:   log likelihood = -13224.823
Iteration 1:   log likelihood = -13085.516
Iteration 2:   log likelihood = -13085.233
Iteration 3:   log likelihood = -13085.233
```

Refining starting values:

```
Grid node 0:   log likelihood = -12893.392
```

Fitting full model:

```
Iteration 0:   log likelihood = -12893.392   (not concave)
Iteration 1:   log likelihood = -12883.273
Iteration 2:   log likelihood = -12874.274
Iteration 3:   log likelihood = -12870.868
Iteration 4:   log likelihood = -12870.814
Iteration 5:   log likelihood = -12870.814
```

```
Mixed-effects ologit regression
Group variable:      state
```

```
Number of obs      =    10,340
Number of groups   =         29
```

```
Obs per group:
      min =         98
      avg =        356.6
      max =        509
```

```
Integration method: mvaghermite
```

```
Integration pts.   =         7
```

```
Log likelihood = -12870.814
```

```
Wald chi2(8)       =        263.27
Prob > chi2        =         0.0000
```

```
-----+-----
      electint |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
```

```

-----+-----
      female |  -0.2111279   .0376188   -5.61   0.000   -0.2848594   -0.1373965
      agecen50 |   .0133066   .0016531    8.05   0.000    .0100666    .0165465
      agecen50sq | -0.0003684   .0000692   -5.32   0.000   -0.000504   -0.0002328
      |
      occtype |
      Other_work | -0.4696381   .0613232   -7.66   0.000   -0.5898293   -0.3494469
      Home | -0.598897   .0942797   -6.35   0.000   -0.7836817   -0.4141122
      Unemployed | -0.8112878   .0910196   -8.91   0.000   -0.989683   -0.6328926
      Retired | -0.5561512   .0784995   -7.08   0.000   -0.7100074   -0.4022949
      Student | -0.0259284   .1070556   -0.24   0.809   -0.2357535   .1838966
-----+-----
      /cut1 | -2.377667   .1103936  -21.54   0.000   -2.594034   -2.161299
      /cut2 | -0.7742671   .1080291   -7.17   0.000   -0.9860001   -0.562534
      /cut3 |  1.452576   .1093899   13.28   0.000    1.238176    1.666977
-----+-----
state |
var(_cons) |  .2311513   .0655831                .1325532   .4030903
-----+-----
LR test vs. ologit model:  chibar2(01) = 428.84          Prob >= chibar2 = 0.0000

```

To store the estimates of this random intercept model:

```
. estimates store ri
```

P9.4.1 Specifying and testing a random slope for age

In this lesson we will consider a random slope model in which the effect of age varies across states. We begin by allowing for state variation in only the linear effect of age. The model can be written as

$$\begin{aligned}
 \text{logit}\{\Pr(y_i > s | \mathbf{x}_{ij})\} \\
 &= \beta_1 \text{female}_{ij} + \beta_2 \text{agecen50}_{ij} + \beta_3 \text{agecen50sq}_{ij} + \beta_4 \text{otherwork}_{ij} \\
 &+ \beta_5 \text{home}_{ij} + \beta_6 \text{unemployed}_{ij} + \beta_7 \text{retired}_{ij} + \beta_8 \text{student}_{ij} + u_{0j} \\
 &+ u_{1j} \text{agecen50}_{ij} - \kappa_s, \quad s = 1, 2, 3
 \end{aligned}$$

$$\begin{pmatrix} u_{0j} \\ u_{1j} \end{pmatrix} \sim N \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{u0}^2 & \\ \sigma_{u01} & \sigma_{u1}^2 \end{pmatrix} \right\}$$

where u_{0j} and u_{1j} denote the state intercept and slope random effects assumed bivariate normally distributed with zero means variances σ_{u0}^2 and σ_{u1}^2 and covariance σ_{u01} .

To allow a covariate to have a random slope, we need to include it to the right of the double separator `||` (after the cluster variable and colon) as well as to the left of the double separator. We must also explicitly request `meologit` to allow the random intercept and slopes effects to correlate. We do this by specifying `unstructured` in the `covariance()` option.:

```
. meologit electint female agecen50 agecen50sq i.occtype ///
|| state: agecen50, covariance(unstructured)
```

Fitting fixed-effects model:

```
Iteration 0: log likelihood = -13224.823
```

```
Iteration 1:    log likelihood = -13085.516
Iteration 2:    log likelihood = -13085.233
Iteration 3:    log likelihood = -13085.233
```

Refining starting values:

```
Grid node 0:    log likelihood =          .
Grid node 1:    log likelihood =          .
Grid node 2:    log likelihood =          .
Grid node 3:    log likelihood =          .
Grid node 4:    log likelihood =          .
Grid node 5:    log likelihood =          .
Grid node 6:    log likelihood =          .
Grid node 7:    log likelihood =          .
Grid node 8:    log likelihood =          .
Grid node 9:    log likelihood =          .
(note: Grid search failed to find values that will yield a log likelihood value.)
```

Fitting full model:

```
initial values not feasible
r(1400);
```

The `meologit` command will fail to fit the model and provide the error report “initial values not feasible”. This is because the model uses a default starting value of 1 for the intercept and slope variance parameters, which fails because one or other parameter in our model is far from the value of 1 (we shall see shortly that it is the variance parameter which is problematic). The `meologit` command then performs a grid search using starting values of 0.1, 1 and 10 for the intercept and slope variances. This also fails, implying that one or other variance lies substantially outside this wide range of values. At this point it is sensible to manually specify a wider grid search. Since it is the introduction of the random-slope which has led to estimation difficulties, we will continue to use a starting value of 1 for the intercept variance and only perform the grid search for the slope variance. We also suspect that given we have measured age in years the age slope effects and their variable will be very small. We therefore re-run the model but specify a lower grid search of the values 0.00001, 0.0001, 0.001 and 0.01 for the slope variance using the option `startgrid()`.

```
. meologit electint female agecen50 agecen50sq i.occtype ///
  || state: agecen50, covariance(unstructured) ///
  startgrid(0.00001 0.0001 0.001 0.01 agecen50[state])
```

Fitting fixed-effects model:

```
Iteration 0:    log likelihood = -13224.823
Iteration 1:    log likelihood = -13085.516
Iteration 2:    log likelihood = -13085.233
Iteration 3:    log likelihood = -13085.233
```

Refining starting values:

```
Grid node 0:    log likelihood =          .
Grid node 1:    log likelihood = -12884.763
Grid node 2:    log likelihood = -12874.035
Grid node 3:    log likelihood = -12891.609
Grid node 4:    log likelihood = -12922.84
```

Fitting full model:

```
Iteration 0:    log likelihood = -12874.035    (not concave)
Iteration 1:    log likelihood = -12862.799    (not concave)
Iteration 2:    log likelihood = -12858.939
```

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```
Iteration 3: log likelihood = -12844.395
Iteration 4: log likelihood = -12844.147
Iteration 5: log likelihood = -12844.144
Iteration 6: log likelihood = -12844.144
```

```
Mixed-effects ologit regression
Group variable: state
```

```
Number of obs = 10,340
Number of groups = 29
```

```
Obs per group:
    min = 98
    avg = 356.6
    max = 509
```

```
Integration method: mvaghermite
```

```
Integration pts. = 7
```

```
Log likelihood = -12844.144
```

```
Wald chi2(8) = 213.08
Prob > chi2 = 0.0000
```

	electint	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
female		-.207515	.0377647	-5.49	0.000	-.2815324	-.1334976
agecen50		.0134811	.0025953	5.19	0.000	.0083945	.0185678
agecen50sq		-.0004009	.0000702	-5.71	0.000	-.0005383	-.0002634
occtype							
Other_work		-.4804009	.061513	-7.81	0.000	-.600964	-.3598377
Home		-.6187383	.0946381	-6.54	0.000	-.8042255	-.433251
Unemployed		-.82943	.0911899	-9.10	0.000	-1.008159	-.6507011
Retired		-.5516042	.0790498	-6.98	0.000	-.7065391	-.3966694
Student		-.0207548	.1075749	-0.19	0.847	-.2315978	.1900881
/cut1		-2.395327	.1128588	-21.22	0.000	-2.616527	-2.174128
/cut2		-.7845681	.1104936	-7.10	0.000	-1.001132	-.5680048
/cut3		1.458201	.1118441	13.04	0.000	1.238991	1.677412
state							
var(agecen50)		.0001125	.0000406			.0000555	.0002282
var(_cons)		.2461234	.0694839			.1415296	.4280146
state							
cov(_cons,agecen50)		.0034604	.0013756	2.52	0.012	.0007642	.0061566

```
LR test vs. ologit model: chi2(3) = 482.18
```

```
Prob > chi2 = 0.0000
```

Note: LR test is conservative and provided only for reference.

The model now runs and displays a slope variance of 0.0001, far lower than the default starting value of 1 or even the default grid search values of 0.1, 1 and 10. We can now store the estimates for the random slope model and run a Likelihood Ratio test to determine if the random slope model provides a better fit to the data than the random intercept model:

```
. estimates store rs
```

```
. lrtest ri rs
```

```
Likelihood-ratio test
(Assumption: ri nested in rs)
```

```
LR chi2(2) = 53.34
Prob > chi2 = 0.0000
```

Note: The reported degrees of freedom assumes the null hypothesis is not on the boundary of the parameter space. If this is not true, then the reported test is conservative.

The test statistic is 53.34 which we compare with a chi-squared distribution on 2 degrees of freedom. The p -value is <0.001 so we can therefore conclude that the (linear) effect of **age** does indeed vary across states.

We now consider whether the quadratic (“flattening off”) effect of age differs across states. We explore this by allowing the coefficient of the age squared term to also vary randomly across countries. The model can be written as

$$\begin{aligned} \text{logit}\{\Pr(y_i > s | \mathbf{x}_{ij})\} \\ = \beta_1 \mathbf{female}_{ij} + \beta_2 \mathbf{agecen50}_{ij} + \beta_3 \mathbf{agecen50sq}_{ij} + \beta_4 \mathbf{otherwork}_{ij} \\ + \beta_5 \mathbf{home}_{ij} + \beta_6 \mathbf{unemployed}_{ij} + \beta_7 \mathbf{retired}_{ij} + \beta_8 \mathbf{student}_{ij} + u_{0j} \\ + u_{1j} \mathbf{agecen50}_{ij} + u_{2j} \mathbf{agecen50sq}_{ij} - \kappa_s, \quad s = 1, 2, 3 \end{aligned}$$

$$\begin{pmatrix} u_{0j} \\ u_{1j} \\ u_{2j} \end{pmatrix} \sim N \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{u0}^2 & & \\ \sigma_{u01} & \sigma_{u1}^2 & \\ \sigma_{u02} & \sigma_{u12} & \sigma_{u2}^2 \end{pmatrix} \right\}$$

Because the estimated slope variance for age in the previous model was about 0.0001 we will try this as a starting value for both the age slope variance σ_{u1}^2 and the new age squared slope variance σ_{u2}^2 in the current model.

```
. meologit electint female agecen50 agecen50sq i.occtype ///
  || state: agecen50 agecen50sq, covariance(unstructured) ///
  startgrid(0.0001 agecen50[state] agecen50sq[state])
```

Fitting fixed-effects model:

```
Iteration 0:   log likelihood = -13224.823
Iteration 1:   log likelihood = -13085.516
Iteration 2:   log likelihood = -13085.233
Iteration 3:   log likelihood = -13085.233
```

Refining starting values:

```
Grid node 0:   log likelihood =          .
Grid node 1:   log likelihood =          .
(note: Grid search failed to find values that will yield a log likelihood value.)
```

Fitting full model:

```
initial values not feasible
r(1400);
```

This model also fails to converge, so we now specify an even lower grid search just for the quadratic age variance:

```
. meologit electint female agecen50 agecen50sq i.occtype ///
  || state: agecen50 agecen50sq, covariance(unstructured) ///
  startgrid(0.0001 agecen50[state]) ///
  startgrid(0.0000001 0.000001 0.00001 agecen50sq[state])
```

Fitting fixed-effects model:

```
Iteration 0:   log likelihood = -13224.823
Iteration 1:   log likelihood = -13085.516
Iteration 2:   log likelihood = -13085.233
Iteration 3:   log likelihood = -13085.233
```

Refining starting values:

```
Grid node 0:   log likelihood =          .
Grid node 1:   log likelihood = -12871.698
Grid node 2:   log likelihood = -12886.004
Grid node 3:   log likelihood = -12915.876
```

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Fitting full model:

```
Iteration 0: log likelihood = -12871.698 (not concave)
Iteration 1: log likelihood = -12867.064 (not concave)
Iteration 2: log likelihood = -12862.519 (not concave)
Iteration 3: log likelihood = -12858.654 (not concave)
Iteration 4: log likelihood = -12852.476
Iteration 5: log likelihood = -12841.391
Iteration 6: log likelihood = -12836.8
Iteration 7: log likelihood = -12836.664
Iteration 8: log likelihood = -12836.663
```

```
Mixed-effects ologit regression      Number of obs   =   10,340
Group variable: state                Number of groups  =    29
```

```
Obs per group:
      min =    98
      avg =   356.6
      max =   509
```

```
Integration method: mvaghermite      Integration pts. =    7
```

```
Wald chi2(8) = 212.05
Prob > chi2 = 0.0000
```

		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
electint							
	female	-.2065826	.0377918	-5.47	0.000	-.2806532	-.1325121
	agecen50	.0133628	.0026321	5.08	0.000	.0082039	.0185216
	agecen50sq	-.0004231	.0000923	-4.59	0.000	-.0006039	-.0002422
	occtype						
	Other_work	-.4916392	.0616455	-7.98	0.000	-.6124623	-.3708162
	Home	-.6463377	.0952191	-6.79	0.000	-.8329638	-.4597117
	Unemployed	-.8391721	.0914013	-9.18	0.000	-1.018315	-.6600288
	Retired	-.5694855	.079449	-7.17	0.000	-.7252028	-.4137683
	Student	-.0286223	.1075251	-0.27	0.790	-.2393677	.1821231
	/cut1	-2.415662	.1185408	-20.38	0.000	-2.647998	-2.183327
	/cut2	-.8023996	.1162716	-6.90	0.000	-1.030288	-.5745115
	/cut3	1.445243	.1175032	12.30	0.000	1.214941	1.675545
state							
	var(agecen50)	.0001165	.0000417			.0000578	.0002348
	var(agecen50sq)	9.48e-08	5.27e-08			3.18e-08	2.82e-07
	var(_cons)	.2820658	.0827954			.1586702	.5014245
state							
	cov(agecen50sq,agecen50)	1.33e-06	1.10e-06	1.21	0.228	-8.32e-07	3.49e-06
	cov(_cons,agecen50)	.0029303	.0014335	2.04	0.041	.0001208	.0057399
	cov(_cons,agecen50sq)	-.0000729	.0000537	-1.36	0.175	-.0001782	.0000324

```
LR test vs. ologit model: chi2(6) = 497.14      Prob > chi2 = 0.0000
```

Note: LR test is conservative and provided only for reference.

The model now converges and we can see that the slope variance for the quadratic age term is tiny: 0.00000009. Three new parameters have been added to the model: Two covariance terms (with the intercept and with the linear age term) and a variance. All three are estimated close to zero, at least to 4 decimal places. It appears that the random slope for the quadratic term is unnecessary, but we will nevertheless carry out a formal test of the null hypothesis that all three parameters are equal to zero:

```
. estimates store rs2
```

```
. lrtest rs rs2
```

```
Likelihood-ratio test      LR chi2(3) = 14.96
(Assumption: rs nested in rs2) Prob > chi2 = 0.0018
```

Note: The reported degrees of freedom assumes the null hypothesis is not on the boundary of the parameter space. If this is not true, then the reported test is conservative.

The test statistic is 14.96 on 3 degrees of freedom, and the p-value is 0.002. We therefore reject the null hypothesis that the three new covariance/variance parameters associated with age-squared are equal to zero, and conclude that the quadratic effect of age does in fact vary significantly across states¹³. We will however now revert to the model without the random coefficient on age-squared in order to make model interpretations simpler. To restore the estimates from the model without a random coefficient for age squared:

```
. estimates restore rs
```

P9.4.2 Interpretation of the random slope model

For a respondent in state j , the effect of age on the log-odds of having a higher (rather than lower) level of interest in EU elections (conditional on gender and employment status) is given by the quadratic function

$$(0.0133 + \hat{u}_{1j})\text{agecen50}_{ij} + 0.0004\text{agecen50sq}_{ij} \quad (9.1)$$

where \hat{u}_{1j} is the state-specific effect of age. The between-state variance in the effect of age is estimated as 0.00011. The intercept variance is estimated as 0.246 which is the between-state variance at age 50 (because age has been centred at 50 years). Note that there has been little change in the intercept variance from the random intercept model (0.231).

To get a better idea of the extent of between-state differences in the effect of age, we will plot the relationship between the log-odds of having a higher election interest and age for each state. To produce this plot we first need to estimate the state residuals \hat{u}_{1j} and substitute into (9.1). We will examine estimates of these slope residuals, and of the intercept residuals, before plotting the relationship with age.

Examining intercept and slope residuals

The positive intercept-slope covariance estimate of 0.0035 implies that states with above-average election interest (intercept residual $\hat{u}_{0j} > 0$) tend also to have a stronger-than-average effect of age (slope residual $\hat{u}_{1j} > 0$). We can calculate the implied correlation in the usual way, by dividing the estimated covariance by the product of the estimated standard deviations.

¹³ This conclusion is different from that obtained in the MLwiN practical because of differences between the Wald test that one is forced to use in MLwiN and the more accurate Likelihood Ratio test that we can employ in Stata.

$$\hat{\rho}_{u01} = \frac{\hat{\sigma}_{u01}}{\sqrt{\hat{\sigma}_{u0}^2} \sqrt{\hat{\sigma}_{u1}^2}}$$

As before, we can refer to the parameter estimates via their internal names.

```
. display _b[cov(_cons[state],agecen50[state]):_cons] / ///
      (sqrt(_b[var(_cons[state]):_cons]))*sqrt(_b[var(agecen50 [state]):_cons])
.00007398
```

The estimated correlation is .00007398.

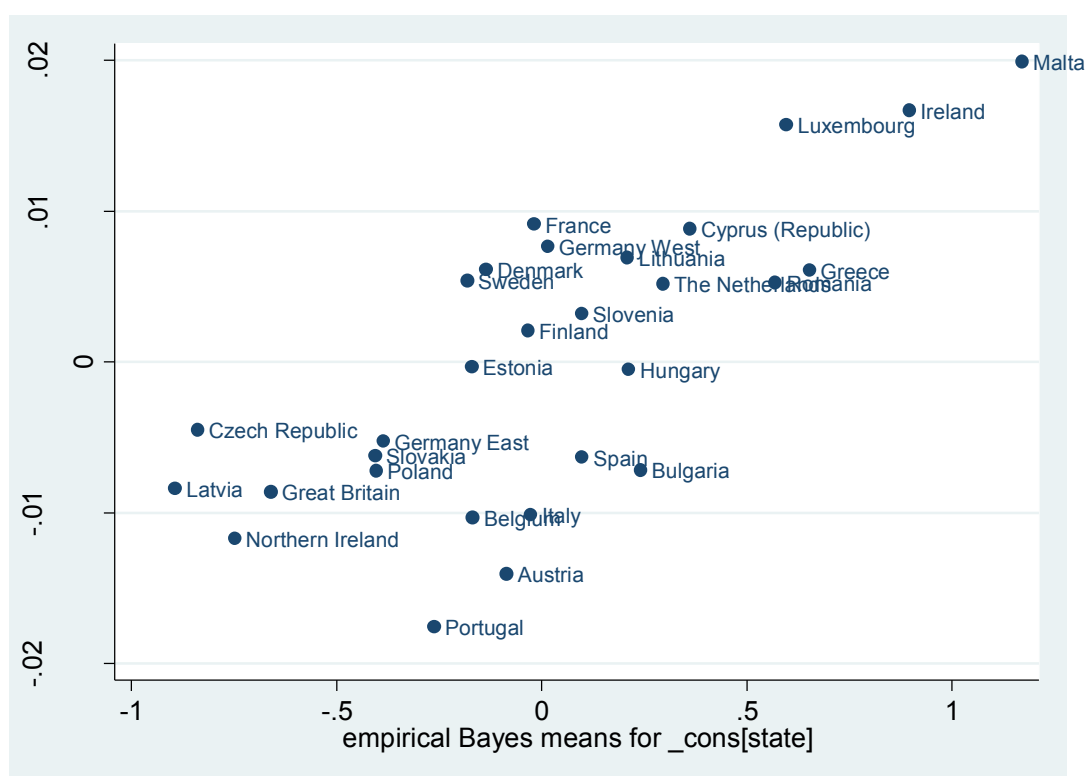
We can store the predicted random intercept and slope effects as before by using the `predict` command with the `reffects` option. Note that the `predict` command will assign the predicted slope effects to the first variable in the variable list and the predicted intercept effects to the second variable in the variable list. We therefore specify the variable list as `u1 u0` rather than `u0 u1`. We also at this point create a binary indicator named `pickone` that identifies each state in the dataset once:

```
. predict u1 u0, reffects
(calculating posterior means of random effects)
(using 7 quadrature points)

. egen pickone = tag(state)
```

To plot the residuals we use the `twoway` command with the option `mlabel(state)` to label each of the states on the plot:

```
. twoway (scatter u1 u0 if pickone==1, mlabel(state))
```



As with the caterpillar plot in P9.4.2, we can see that Malta is the most extreme state at the top right of the graph; the high intercept for Malta implies a high level of election interest¹⁴. The relatively large slope residual, combined with the fixed effect for age ($agecen50 = 0.013$), implies that Malta has a stronger-than-average age effect on the outcome: the difference in election interest between young and old respondents is larger in Malta than in other EU states. The state with the weakest age effect is Portugal.

State prediction curves

The part of the fitted cumulative logit regression equation that captures the relationship between the log-odds of lower election interest and age is

$$(0.0133 + \hat{u}_{1j})agecen50_{ij} + 0.0004agecen50sq_{ij}$$

The state residual on the linear age term, estimated as \hat{u}_{1j} , allows the relationship to vary across states.

As the age effect does not depend on the other predictors, gender and occupation type, we will fix these at their reference categories for convenience. Similarly, as the age effect is assumed to be proportional, we will plot only the log-odds of being above the very low interest category (versus very low).

¹⁴ Note that the plot is flipped from the one obtained in the MLwiN practical because of the different category order used for `electint`

To produce a plot of the predicted state lines, we need to first compute the predicted log-odds of having above very low interest in elections for each individual based on their age and their state of residence, and then plot these computed values. To identify the internal parameter names that Stata uses for each coefficient we rerun the `meologit` command with the option `coeflegend`:

```
. meologit, coeflegend
```

```
Mixed-effects ologit regression                                Number of obs      =      10,340
Group variable:      state                                    Number of groups   =         29

                                           Obs per group:
                                           min =             98
                                           avg =          356.6
                                           max =           509

Integration method: mvaghermite                               Integration pts.   =         7

Log likelihood = -12844.144                                   Wald chi2(8)      =      213.08
                                                                Prob > chi2       =      0.0000
```

	electint	Coef.	Legend
	female	-.207515	_b[electint:female]
	agecen50	.0134811	_b[electint:agecen50]
	agecen50sq	-.0004009	_b[electint:agecen50sq]
	occtype		
	Other_work	-.4804009	_b[electint:2.occtype]
	Home	-.6187383	_b[electint:3.occtype]
	Unemployed	-.82943	_b[electint:4.occtype]
	Retired	-.5516042	_b[electint:5.occtype]
	Student	-.0207548	_b[electint:6.occtype]
	/cut1	-2.395327	_b[cut1:_cons]
	/cut2	-.7845681	_b[cut2:_cons]
	/cut3	1.458201	_b[cut3:_cons]
state			
	var(agecen50)	.0001125	_b[var(agecen50[state]):_cons]
	var(_cons)	.2461234	_b[var(_cons[state]):_cons]
state			
	cov(_cons,agecen50)	.0034604	_b[cov(_cons[state],agecen50[state]):_cons]

```
LR test vs. ologit model: chi2(3) = 482.18                      Prob > chi2 = 0.0000
```

Note: LR test is conservative and provided only for reference.

The internal parameter names are listed under `Legend`. We see that the internal parameter name for referring to the estimated coefficient of 0.0134811 for **agecen50** is `_b[electint:agecen50]`. Similarly, the estimated coefficient of -0.0004009 for **agecen50sq** can be referred to by `_b[electint:agecen50sq]` while the first cut point can be referred to by `_b[cut1:_cons]`.

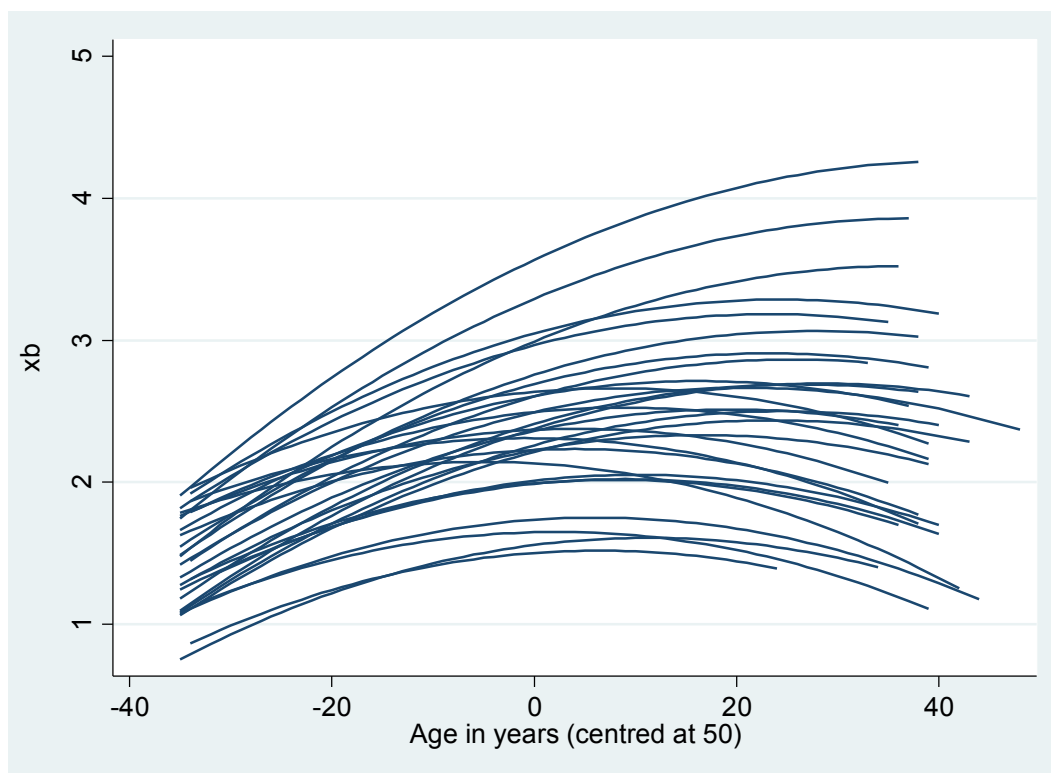
We now compute the predicted log odds of having low, some or very high interest in EU elections and store them in a new variable **xb**. We also at this point sort the dataset on **state** and **agecen50**:

$$\hat{\beta}_2 \text{agecen50}_{ij} + \hat{\beta}_3 \text{agecen50sq}_{ij} + \hat{u}_{0j} + \hat{u}_{1j} \text{agecen50}_{ij} - \kappa_1$$

```
. generate xb = _b[electint:agecen50]*agecen50 ///
+ _b[electint:agecen50sq]*agecen50^2 + u0 + u1*agecen50 - _b[cut1:_cons]
. sort state agecen50
```

To plot the predicted state election interest by age curves we again use the `twoway` command, but this time with the option `connect(ascending)` to connect the predicted points of each of the states in ascending order:

```
. twoway (line xb agecen50, connect(ascending))
```



Notice that the curves are ‘fanning out’ as age increases, which is expected because of the positive estimate of the intercept-slope covariance.

Between-state variance as a function of age

From the plot of the predicted curves for each community, we can see that the curves are more spread out vertically at higher ages than at younger ages: the variability in the log-odds of having higher interest in EU elections increases as age increases. Fitting a random slope for age implies that the between-state variance is a function of age, rather than constant as in the random intercept model. The between-state variance takes the following form:

$$\begin{aligned}\text{Var}(u_{0j} + u_{1j}\text{age}_{ij}) &= \text{Var}(u_{0j}) + 2\text{Cov}(u_{0j}, u_{1j})\text{age}_{ij} + \text{Var}(u_{1j})\text{age}_{ij}^2 \\ &= \sigma_{u0}^2 + 2\sigma_{u01}\text{age}_{ij} + \sigma_{u1}^2\text{age}_{ij}^2\end{aligned}$$

Where u_{0j} is the state intercept effect, u_{1j} is the state slope effect, σ_{u0}^2 is the intercept variance, σ_{u1}^2 is the slope variance, and σ_{u01} is the intercept-slope covariance. Substituting in the parameter estimates gives

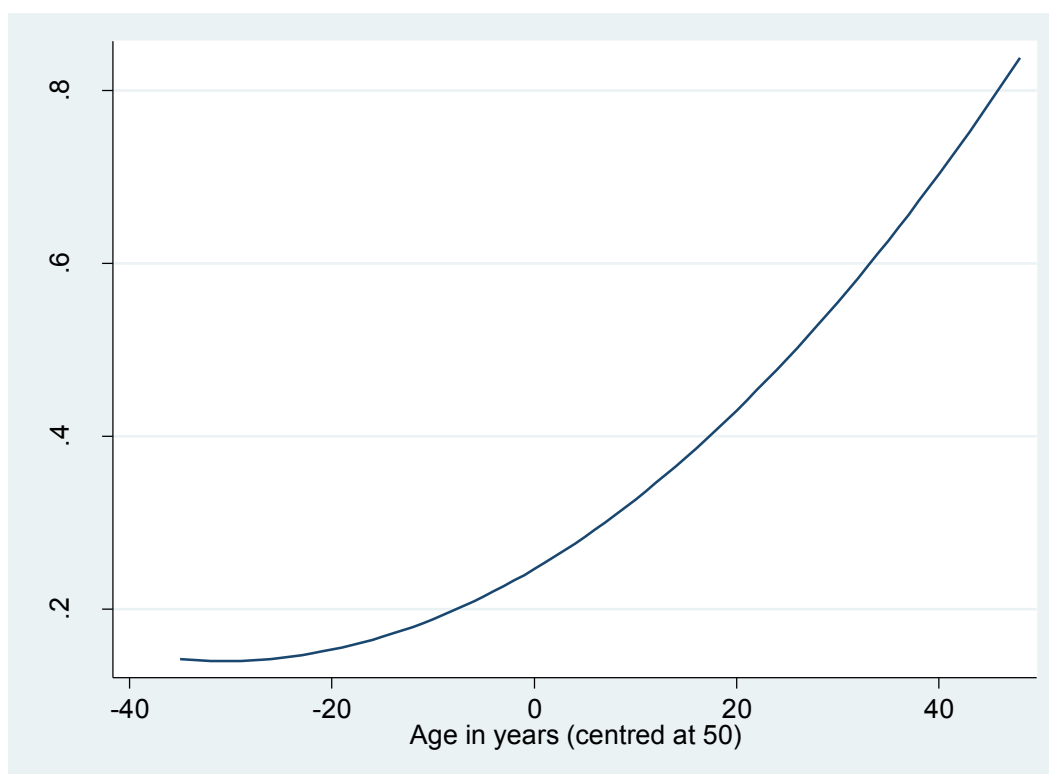
$$0.246 + 0.007\text{age}_{ij} + 0.001\text{age}_{ij}^2$$

We can calculate this state-level variance function by referring to the estimated model parameters by their internal parameter names.

```
. generate lev2var = _b[var(_cons[state]):_cons] ///
+ 2*_b[cov(_cons[state],agecen50[state]):_cons]*agecen50 ///
+ _b[var(agecen50[state]):_cons]*agecen50^2
```

We can now plot the between-state variance:

```
. twoway (line lev2var agecen50, sort)
```




As expected from the ‘fanning out’ pattern of the state prediction curves, the between-state variance increases as a function of age.

P9.5 Contextual Effects

We have so far considered only level 1 explanatory variables, i.e. individual characteristics. In this lesson, we consider the effect of a level 2 variable: the proportion of respondents in a state that live in a rural area. Specifically, we will explore whether there is an effect of state-level rurality on election interest that is over and above any effect of a respondent’s type of community of residence.

Load “9.5.dta” into memory and if it is not already in use open the do-file “9.5.do” for this lesson.

From within the LEMMA Learning Environment

- Go to **Module 9: Single-Level and Multilevel Models for Ordinal Responses**, and scroll down to  **Stata datasets and dofiles** Click “9.5.dta” to open the dataset

Refit the random slope model fitted in P9.4:

```
. meologit electint female agecen50 agecen50sq i.occtype ///
> || state: agecen50, covariance(unstructured) ///
> startgrid(0.0001 agecen50[state])
```

Fitting fixed-effects model:

```
Iteration 0: log likelihood = -13224.823
Iteration 1: log likelihood = -13085.516
Iteration 2: log likelihood = -13085.233
Iteration 3: log likelihood = -13085.233
```

Refining starting values:

```
Grid node 0: log likelihood = .
Grid node 1: log likelihood = -12874.035
```

Fitting full model:

```
Iteration 0: log likelihood = -12874.035 (not concave)
Iteration 1: log likelihood = -12862.799 (not concave)
Iteration 2: log likelihood = -12858.939
Iteration 3: log likelihood = -12844.395
Iteration 4: log likelihood = -12844.147
Iteration 5: log likelihood = -12844.144
Iteration 6: log likelihood = -12844.144
```

```
Mixed-effects ologit regression      Number of obs      =      10,340
Group variable: state                 Number of groups    =         29
```

```
Obs per group:
      min =         98
      avg =       356.6
      max =        509
```

```
Integration method: mvaghermite      Integration pts.    =         7
```

```
Wald chi2(8)      =      213.08
Prob > chi2       =      0.0000
Log likelihood = -12844.144
```

	electint	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
female		-.207515	.0377647	-5.49	0.000	-.2815324	-.1334976
agecen50		.0134811	.0025953	5.19	0.000	.0083945	.0185678
agecen50sq		-.0004009	.0000702	-5.71	0.000	-.0005383	-.0002634
occtype							
Other_work		-.4804009	.061513	-7.81	0.000	-.600964	-.3598377
Home		-.6187383	.0946381	-6.54	0.000	-.8042255	-.433251
Unemployed		-.82943	.0911899	-9.10	0.000	-1.008159	-.6507011
Retired		-.5516042	.0790498	-6.98	0.000	-.7065391	-.3966694
Student		-.0207548	.1075749	-0.19	0.847	-.2315978	.1900881
/cut1		-2.395327	.1128588	-21.22	0.000	-2.616527	-2.174128
/cut2		-.7845681	.1104936	-7.10	0.000	-1.001132	-.5680048

```

-----+-----
          /cut3 |      1.458201   .1118441   13.04   0.000   1.238991   1.677412
-----+-----
state
  var(agecen50) |      .0001125   .0000406                .0000555   .0002282
  var(_cons)   |      .2461234   .0694839                .1415296   .4280146
-----+-----
state
cov(_cons,agecen50) |      .0034604   .0013756   2.52   0.012   .0007642   .0061566
-----+-----
LR test vs. ologit model: chi2(3) = 482.18                Prob > chi2 = 0.0000

```

Note: LR test is conservative and provided only for reference.

The dataset contains a variable called **commtype** which classifies a respondent's community of residence (coded as 1=rural, 2=mid-sized town, 3=large town or city). We will first view a one-way tabulation of this variable:

```

. tabulate commtype

   Type of |
community |      Freq.      Percent      Cum.
-----+-----
   rural   |      3,803      36.78      36.78
 mid_town  |      3,722      36.00      72.78
 big_town  |      2,815      27.22     100.00
-----+-----
      Total |     10,340     100.00

```

We can see that there is a relatively even proportion of respondents living in each of the community types across the sample. We now add **commtype** to the model, taking rural as the reference category. The model can be written as

$$\begin{aligned}
 &\text{logit}\{\Pr(y_i > s | \mathbf{x}_{ij})\} \\
 &= \beta_1 \mathbf{female}_{ij} + \beta_2 \mathbf{agecen50}_{ij} + \beta_3 \mathbf{agecen50sq}_{ij} + \beta_4 \mathbf{otherwork}_{ij} \\
 &+ \beta_5 \mathbf{home}_{ij} + \beta_6 \mathbf{unemployed}_{ij} + \beta_7 \mathbf{retired}_{ij} + \beta_8 \mathbf{student}_{ij} \\
 &+ \beta_9 \mathbf{midtown}_j + \beta_{10} \mathbf{bigtown}_j + u_{0j} + u_{1j} \mathbf{agecen50}_{ij} - \kappa_s, \\
 &s = 1, 2, 3
 \end{aligned}$$

$$\begin{pmatrix} u_{0j} \\ u_{1j} \end{pmatrix} \sim N \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{u0}^2 & \\ \sigma_{u01} & \sigma_{u1}^2 \end{pmatrix} \right\}$$

Fitting the model gives the following results.

```

. meologit electint female agecen50 agecen50sq i.occtype ///
> i.commtype ///
> || state: agecen50, covariance(unstructured) ///
> startgrid(0.0001 agecen50[state])

```

Fitting fixed-effects model:

```

Iteration 0:  log likelihood = -13224.823
Iteration 1:  log likelihood = -13072.587
Iteration 2:  log likelihood = -13072.25
Iteration 3:  log likelihood = -13072.25

```

Refining starting values:

```

Grid node 0:  log likelihood = .
Grid node 1:  log likelihood = -12865.565

```

Fitting full model:

Module 9 (Stata Practical): Single-level and Multilevel Models for Ordinal Responses

```

Iteration 0:  log likelihood = -12865.565   (not concave)
Iteration 1:  log likelihood = -12854.264   (not concave)
Iteration 2:  log likelihood = -12850.426
Iteration 3:  log likelihood = -12834.22
Iteration 4:  log likelihood = -12834.088
Iteration 5:  log likelihood = -12834.086
Iteration 6:  log likelihood = -12834.086

Mixed-effects ologit regression
Group variable:      state

Number of obs      =      10,340
Number of groups   =         29

Obs per group:
    min =          98
    avg =        356.6
    max =         509

Integration method: mvaghermite
Integration pts.   =          7

Log likelihood = -12834.086
Wald chi2(10)     =      233.15
Prob > chi2       =      0.0000

```

	electint	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
female		-.2122067	.0378001	-5.61	0.000	-.2862935 - .1381199
agecen50		.0136729	.002575	5.31	0.000	.0086259 .0187199
agecen50sq		-.0004121	.0000702	-5.87	0.000	-.0005496 -.0002745
occtype						
Other_work		-.4588217	.0617163	-7.43	0.000	-.5797835 -.3378599
_Home		-.5954577	.0948074	-6.28	0.000	-.7812769 -.4096385
Unemployed		-.8006215	.0914324	-8.76	0.000	-.9798256 -.6214174
Retired		-.5299013	.0792293	-6.69	0.000	-.6851878 -.3746148
Student		-.0044276	.1076669	-0.04	0.967	-.2154509 .2065957
commttype						
mid_town		.1250435	.0435274	2.87	0.004	.0397313 .2103556
big_town		.207336	.0475058	4.36	0.000	.1142263 .3004456
/cut1		-2.283703	.1163453	-19.63	0.000	-2.511735 -2.05567
/cut2		-.6711383	.1141836	-5.88	0.000	-.894934 -.4473425
/cut3		1.574393	.1156923	13.61	0.000	1.34764 1.801145
state						
var(agecen50)		.0001094	.0000397			.0000538 .0002227
var(_cons)		.2501873	.0706975			.1437918 .4353075
state						
cov(_cons,agecen50)		.0034614	.0013757	2.52	0.012	.0007652 .0061577

```

LR test vs. ologit model: chi2(3) = 476.33
Prob > chi2 = 0.0000

```

Note: LR test is conservative and provided only for reference.

Living in a more urban area is associated with a higher probability of being in a high response category (i.e. higher interest in EU elections). Equivalently, we can say that living in rural areas is associated with a lower level of interest in EU elections. Interest is highest in large towns or cities, and lowest in rural areas.

We now consider whether there is an effect on election interest of the extent of urbanisation in a state, which we measure by the state-level proportion of respondents who live in rural communities. By including this state-level measure in addition to type of community of residence, we can investigate the following questions: Does the urbanisation of a state as a whole affect their interest in EU elections (perhaps indirectly through an effect on a person's engagement with EU matters)? Or does any effect operate only at a local, community level?

We first need to create a dummy variable named **rural** for rural community of residence, coded to 1 if the respondent lives in a rural community, 0 otherwise:

```
. generate rural = (commtype==1)
```

We now aggregate **rural** to the state level. We will compute the state-level mean of **rural** in a new variable called **staterural**, which for a binary variable is equal to the state-level proportion with **rural**=1. We then create a binary indicator **pickone** that, as before, identifies each state in the dataset once:

```
. bysort state: egen staterural = mean(rural)
```

```
. egen pickone = tag(state)
```

Next we list the states and their mean values of **staterural** in ascending order:

```
. sort staterural
```

```
. list state staterural if pickone==1
```

	state	stater~1
65.	Germany East	.1825726
430.	Italy	.2117647
844.	Finland	.2387097
1222.	Lithuania	.2447552
1641.	Great Britain	.2564103
1960.	Denmark	.2755102
2322.	Cyprus (Republic)	.2931035
2603.	Hungary	.2979798
3032.	Germany West	.3137255
3312.	Bulgaria	.3172804
3654.	Estonia	.3233696
4009.	Northern Ireland	.3257576
4332.	Greece	.3273196
4610.	Latvia	.3661017
4836.	Poland	.3774194
5575.	The Netherlands	.38125
5924.	Ireland	.4022039
6264.	Portugal	.4069401
6729.	Sweden	.4125737
6917.	Czech Republic	.4125874
7537.	Spain	.4155496
7773.	Slovenia	.4285714
7988.	Romania	.444079
8307.	Slovakia	.4484305
8873.	Luxembourg	.4848485
9147.	Austria	.4926471
9366.	France	.4933333
10110.	Belgium	.5184382
10332.	Malta	.6836734

We see that the proportion rural in a state ranges from 0.18 in East Germany to 0.68 in Malta.

We will now add **staterural** to the model.

$$\begin{aligned} \text{logit}\{\Pr(y_i > s | \mathbf{x}_{ij})\} \\ = \beta_1 \text{female}_{ij} + \beta_2 \text{agecen50}_{ij} + \beta_3 \text{agecen50sq}_{ij} + \beta_4 \text{otherwork}_{ij} \\ + \beta_5 \text{home}_{ij} + \beta_6 \text{unemployed}_{ij} + \beta_7 \text{retired}_{ij} + \beta_8 \text{student}_{ij} \\ + \beta_9 \text{midtown}_j + \beta_{10} \text{bigtown}_j + \beta_{11} \text{staterural}_j + u_{0j} + u_{1j} \text{agecen50}_{ij} \\ - \kappa_s, \quad s = 1, 2, 3 \end{aligned}$$

$$\begin{pmatrix} u_{0j} \\ u_{1j} \end{pmatrix} \sim N \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{u0}^2 & \\ & \sigma_{u1}^2 \end{pmatrix} \right\}$$

```
. meologit electint female agecen50 agecen50sq i.occtype ///
  i.commtype staterural ///
  || state: agecen50, covariance(unstructured) ///
  startgrid(0.0001 agecen50[state])
```

Fitting fixed-effects model:

```
Iteration 0: log likelihood = -13224.823
Iteration 1: log likelihood = -13064.765
Iteration 2: log likelihood = -13064.391
Iteration 3: log likelihood = -13064.391
```

Refining starting values:

```
Grid node 0: log likelihood = .
Grid node 1: log likelihood = -12863.903
```

Fitting full model:

```
Iteration 0: log likelihood = -12863.903 (not concave)
Iteration 1: log likelihood = -12853.233 (not concave)
Iteration 2: log likelihood = -12849.144
Iteration 3: log likelihood = -12845.321
Iteration 4: log likelihood = -12832.444
Iteration 5: log likelihood = -12832.408
Iteration 6: log likelihood = -12832.408
```

```
Mixed-effects ologit regression      Number of obs      =      10,340
Group variable: state                 Number of groups    =         29
```

```
Obs per group:
      min =         98
      avg =       356.6
      max =        509
```

```
Integration method: mvaghermite      Integration pts.    =         7
```

```
Wald chi2(11)      =      237.08
Prob > chi2        =      0.0000
Log likelihood = -12832.408
```

electint	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
female	-.2127722	.0377984	-5.63	0.000	-.2868557	-.1386887
agecen50	.013633	.0025451	5.36	0.000	.0086445	.0186214
agecen50sq	-.0004138	.0000702	-5.89	0.000	-.0005513	-.0002762
occtype						
Other_work	-.4589917	.0617039	-7.44	0.000	-.5799292	-.3380543
Home	-.59577	.0947927	-6.28	0.000	-.7815603	-.4099797
Unemployed	-.8001363	.091419	-8.75	0.000	-.9793143	-.6209583
Retired	-.530029	.0792184	-6.69	0.000	-.6852942	-.3747638
Student	-.0052502	.1076691	-0.05	0.961	-.2162779	.2057774
commtype						
mid_town	.1293679	.0435771	2.97	0.003	.0439584	.2147775

```

      big_town |      .2121747      .0475821      4.46      0.000      .1189156      .3054339
      staterural |      1.384934      .743873      1.86      0.063      -.0730307      2.842898
-----+-----
      /cut1 |     -1.768164      .2979694     -5.93      0.000     -2.352173     -1.184155
      /cut2 |     -.1556247      .2973999     -0.52      0.601     -.7385178      .4272683
      /cut3 |      2.090016      .2983435      7.01      0.000      1.505274      2.674759
-----+-----
state
      var(agecen50) |      .000105      .0000383
      var(_cons) |      .2208761      .0623483
-----+-----
state
      cov(_cons,agecen50) |      .0030955      .0012653      2.45      0.014      .0006156      .0055754
-----+-----
LR test vs. ologit model: chi2(3) = 463.97                      Prob > chi2 = 0.0000

```

Note: LR test is conservative and provided only for reference.

We find that the effects of type of community (**mid_town** and **big_town**) are almost unchanged by the addition of the proportion rural in a state, and remain statistically significant. The Z-ratio for **state_rural** is $1.385/0.744 = 1.86$ which is close to significance at the 5% level. The positive coefficient for **state_rural** indicates that living in a more rural state is associated with greater interest in EU elections.¹⁵ These results suggest that, interestingly, the effect of urbanisation (or rurality) at the state level is in the opposite direction to its effect at the local level.

¹⁵ Or a higher probability of being in a high interest group.